Mathematical Girls

story by Hiroshi Yuki　comic by Mika Hisaka

下

聞かせてよ
君の答えを——

[原作]結城 浩
[作画]日坂水柯

数学を通じて「わからない」と向きあう男の子女の子
世界初数学コミック感動のクライマックス!

数学ガール

MFコミックス

数学ガール

数学×青春

[原作]結城 浩
[作画]日坂水柯

下

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メディアファクトリー
MEDIA FACTORY

「僕」の憧れ、ミルカさん。数式をこよなく愛し、
数学についてだけ饒舌な黒髪メガネの才媛。
ふたりで数式と親しむ間に狭まった距離。
ふたりで作った半径ゼロの円。
キスを交わしたのにいつもと同じ関係の二人。
「僕」を慕う、後輩の女の子テトラちゃん。
ちっちゃくてグラマーで妹みたいな彼女は
よりいっそう「僕」への気持ちを強くする…。
交わりあう3人の気持ち。時を超える数学のメッセージ。
みなで解くフィボナッチ数列の一般項。
さあ答えを出そう!

「あたし 今 口をひらくと
あらぬことまで言ってしまいそうで……」
——これ以上は言葉になんか、しない

「僕」への気持ちを胸に秘めたテトラちゃん。
ミルカさんを見つめ続ける「僕」。
ミルカさんは何を見据えているのだろう。
数学も、この想いも、時を超えてゆく——

最先端数学題材の
数学小説
コミカライズ、
堂々の完結!

結城 浩

1963年生まれ。プログラム書きと文章書きが趣味&仕事。著書は『数学ガール』シリーズの他、『プログラマの数学』『増補改訂版Java言語で学ぶデザインパターン入門』『新版暗号技術入門』など。パッハのフーガが好きなプロテスタントのクリスチャン。

http://www.hyuki.com/

日坂水柯

6月生まれA型。好きなTV番組→NHK高校講座地学。将来の夢→山種美術館の近くに住んで通いまくること。

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フラッパー・ドットコム好評稼動中!!

数学ガール

[illegible]

[illegible]柯

「数学ガール」下・日本語

Through mathematics, we try to confront the "unknown."
Despite this,

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あとがき（結城 浩） 186

この物語はフィクションであり
実在する人物・団体とは関係ありません

At times,

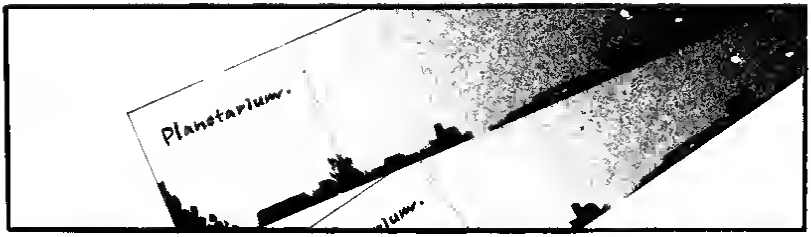
Inexplicably,

[原作]結城 浩
（『数学ガール』ソフトバンククリエイティブ刊）
[作画]日坂水柯

数学ガール
Mathematical Girls 下

story by Hiroshi Yuki comic by Hika Hisaka

We find ourselves
at a loss.


Planetarium.

Here.
lanetarium.

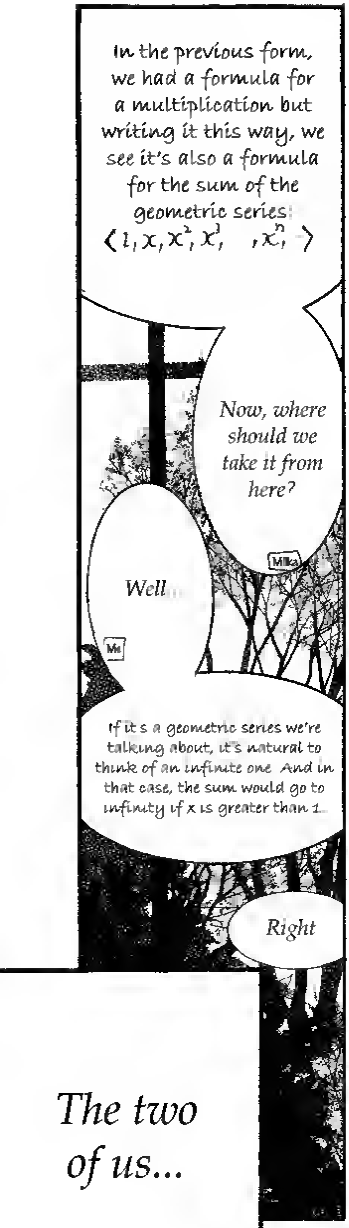
I received a lot of tickets for free,
So take some if you want.
Chapter 8

The planetarium, huh?
Do you want try going there after school?
...It's time to go look for patterns

LIBRARY
Hmm?
Planet-arium tickets?
Yeah, let's go
The two of us, alone.
Let's start off with (1+x)(1-x)
Milka and I

Just like that time...
You're a point on a circle, aren't you?
A unit circle, perhaps...
But that's only if,
The sum of our arms' length is 1 as well

And at that moment,


In the previous form, we had a formula for a multiplication but writing it this way, we see it's also a formula for the sum of the geometric series: ⟨1, x, x², x³, , xⁿ, ⟩
Now, where should we take it from here?
Well...
Mika
Me
If it's a geometric series we're talking about, it's natural to think of an infinite one. And in that case, the sum would go to infinity if x is greater than 1.
Right
The two of us...

Start out by expanding $(1+x)(1-x)$.

$$(1+x)(1-x) = (1+x)\cdot 1 - (1+x)\cdot x$$
$$= (1+x) - (x+x^2)$$
$$= 1 + (x-x) - x^2$$
(cancels out)
$$= 1 - x^2$$

Next, try expanding $(1+x+x^2)(1-x)$ instead.

$$(1+x+x^2)(1-x) = (1+x+x^2)\cdot 1 - (1+x+x^2)\cdot x$$
$$= (1+x+x^2) - (x+x^2+x^3)$$
$$= 1 + (x-x) + (x^2-x^2) - x^3$$
(cancels out)
$$= 1 - x^3$$

The pattern is obvious. Only the first and last terms remain while the others all cancel out. This is easier to see if we use the standard long multiplication method instead, which will be demonstrated below in the case of $(1+x+x^2+x^3)(1-x)$.

$$\begin{array}{r} 1 + x + x^2 + x^3 \\ \times \quad 1 - x \\ \hline -x - x^2 - x^3 - x^4 \\ 1 + x + x^2 + x^3 \\ \hline 1 \qquad\qquad - x^4 \end{array}$$

If we multiply x to the power of n, the general form is as below

$$(1)(1-x) = 1 - x^1$$
$$(1+x)(1-x) = 1 - x^2$$
$$(1+x+x^2)(1-x) = 1 - x^3$$
$$(1+x+x^2+x^3)(1-x) = 1 - x^4$$
$$(1+x+x^2+x^3+x^4)(1-x) = 1 - x^5$$
$$(1+x+x^2+x^3+x^4+\cdots+x^n)(1-x) = 1 - x^{n+1}$$

Easy enough so far, right? Where should we go from here? Well, start by writing the previous again

$$(1+x+x^2+x^3+x^4+\cdots+x^n)(1-x) = 1 - x^{n+1}$$

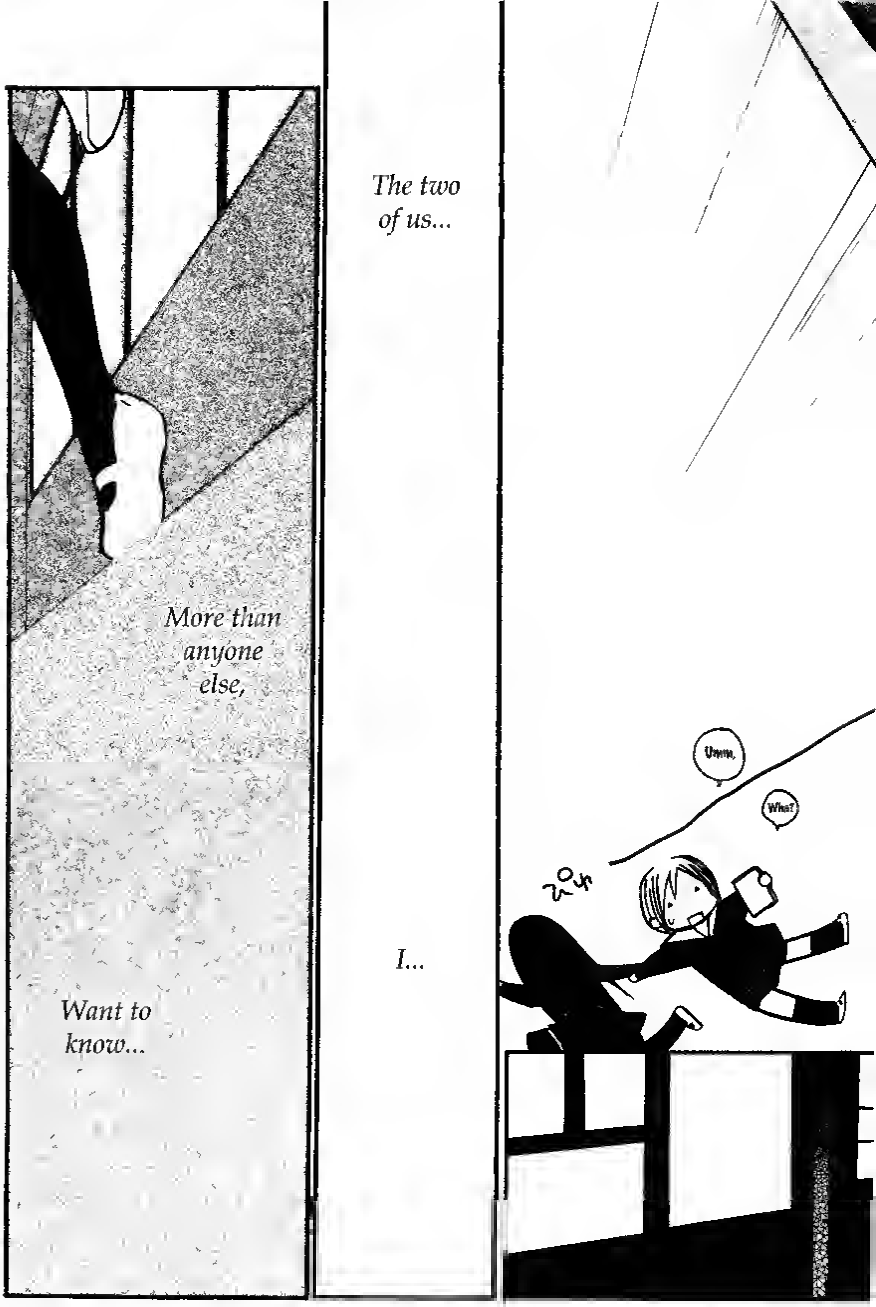
Now, assuming that $1-x \neq 0$, divide both sides by $1-x$ to obtain the following.

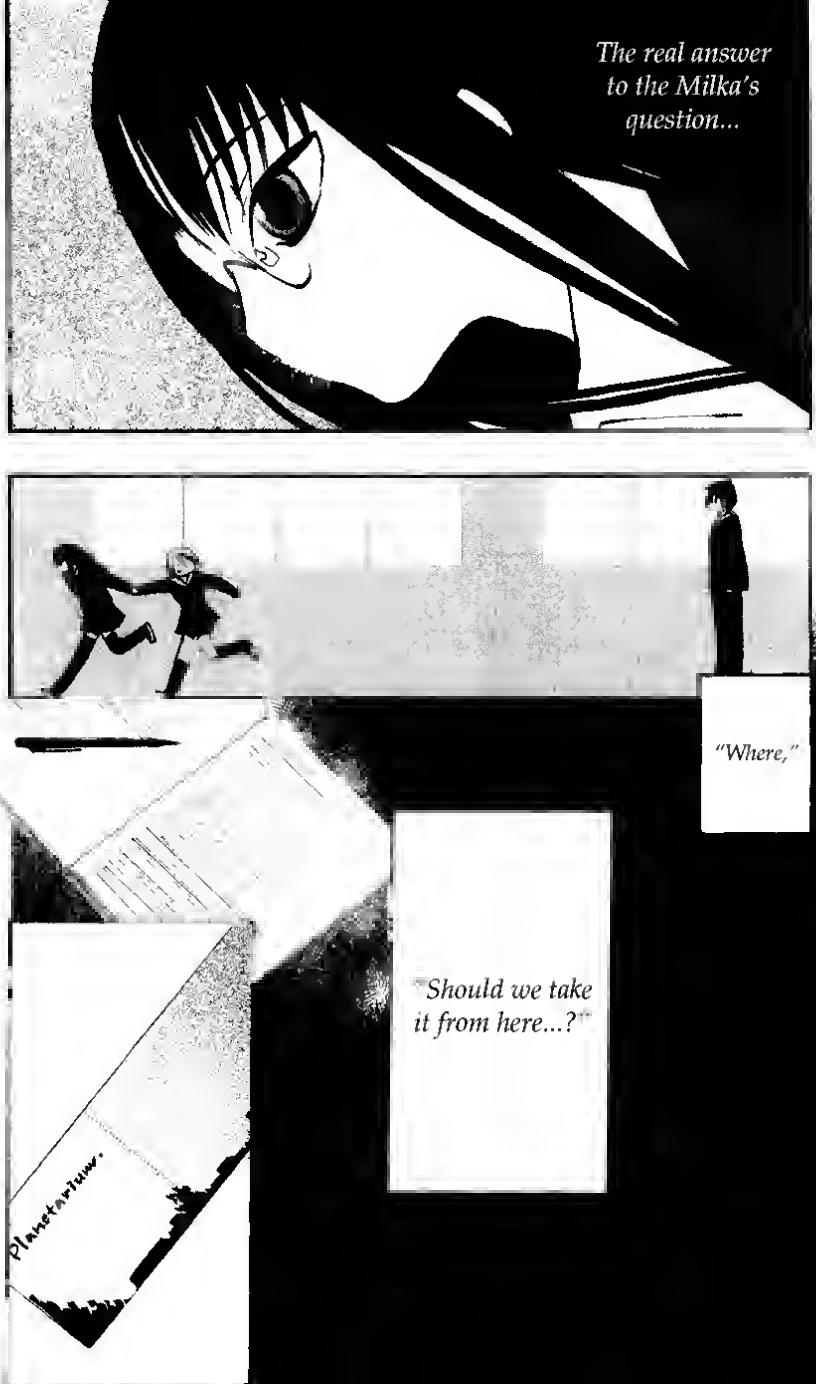
$$1 + x + x^2 + x^3 + x^4 + \cdots + x^n = \frac{1 - x^{n+1}}{1-x}$$

.....
Library Staff
Excuse me but it's closing time
Should we continue somewhere else?
Ah!
Senpai!

Tetra?
I know it's after school hours,
But I thought you might be here so I-
Umm
.....

ぶわっ
ぐ
ぐ
ぐ


More than anyone else,
Want to know...
The two of us...
I...
Umm.
What?


The real answer to the Milka's question...
"Where,"
"Should we take it from here...?"
Planetarium.

T I P S

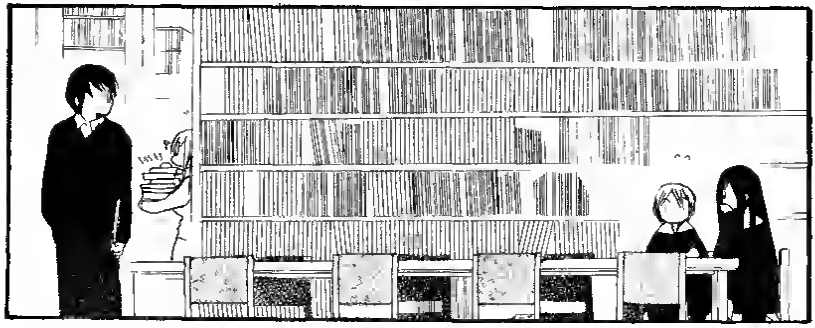
[*The Library After Schaal*]

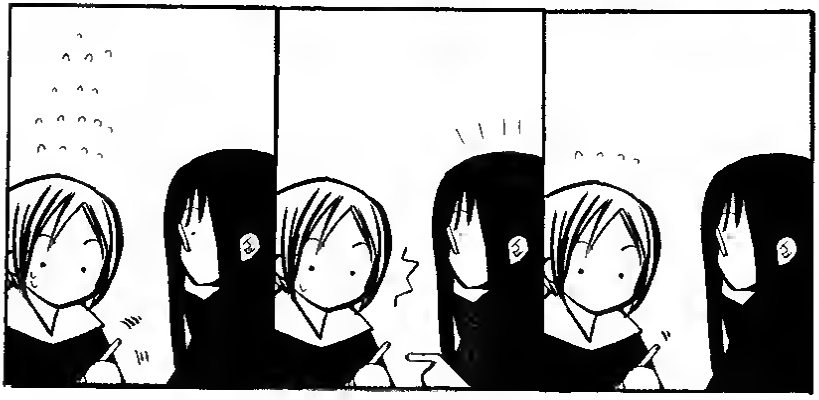
The intellectual place ta be far a student, with an atmasphere quite distinct than that af the hame ar the classraam. Milka and Tetra study here aften in the stary, but the library isn't used exclusively as a place af study alane. On a deeper level, it is alsa a place af discaurse between the authar and the reader. Far these reasans, the library is an all-impartant setting that draws in many fans.

数学ガール

Mathematical Girls

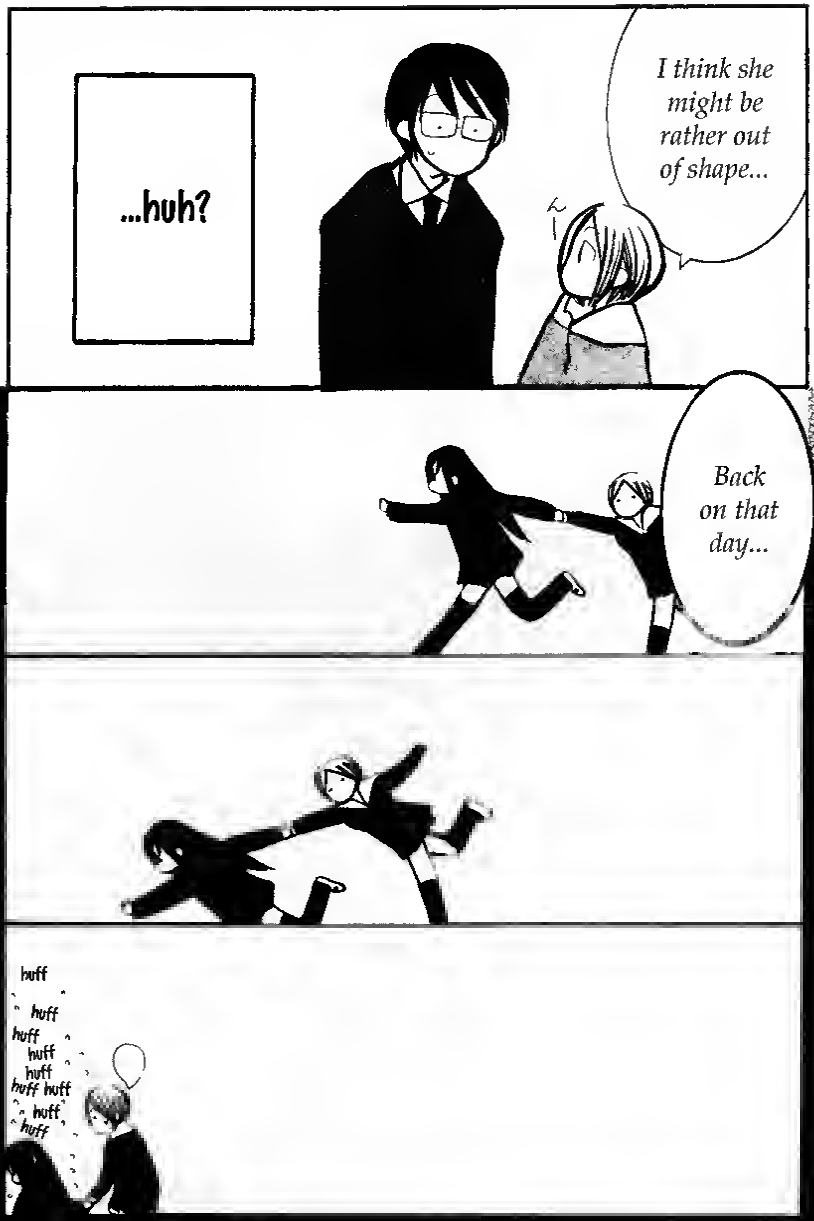
LIBRARY





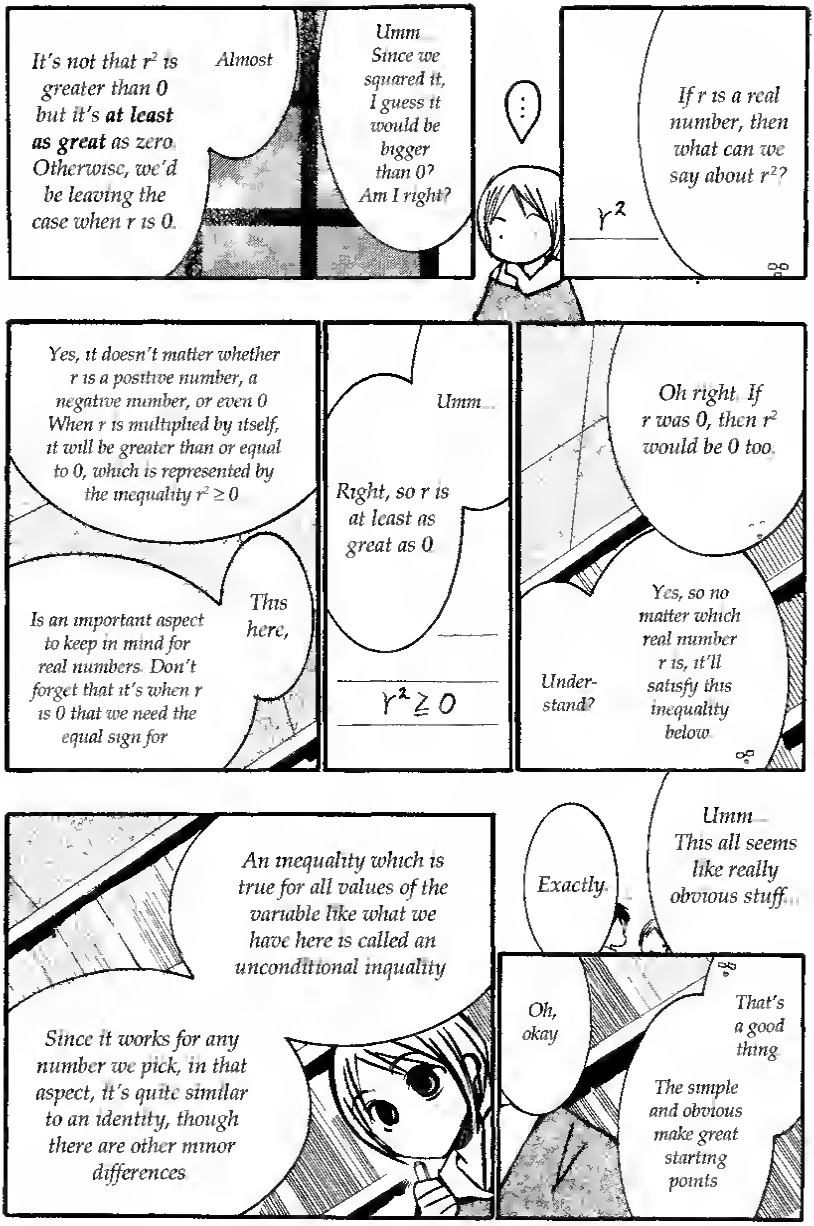
If it's just studying together,
You were there in the library too yesterday?
You could've at least said hi then.
Then I suppose it's fine...
What?

R-
Right, I guess I should have...
.....
Ah-
Speaking of which,
About Milka...
ﾌﾞﾙｰ.

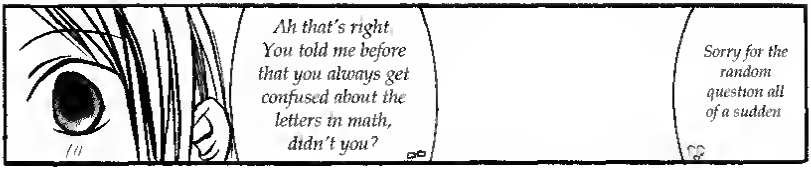

...huh?
I think she might be rather out of shape...
Back on that day...
huff
huff
huff
huff
huff
huff huff
huff
huff

Well, Milka's rather poor on the athletic side of things, you see...
Wha-
Really!?
I never would've expected that...
I mean,
She's so thin and all...
I wonder if she's eating properly?
Hmm. I've only ever seen her eat a little bit of chocoloate...

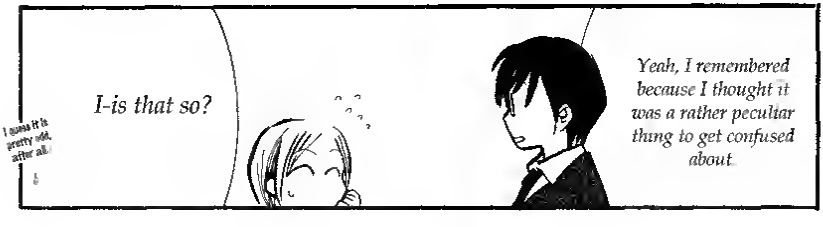
drool
Chocolate?
Hm?
But it's not like you're overweight or anything...
Saying something like this to a woman
?
Sorry, it's just that I'm in the middle of a diet right now...
Uhh... Is that so?
Oh yes, I am! I need to lose weight badly!
Girls...
Is inevitably met with a reply like this

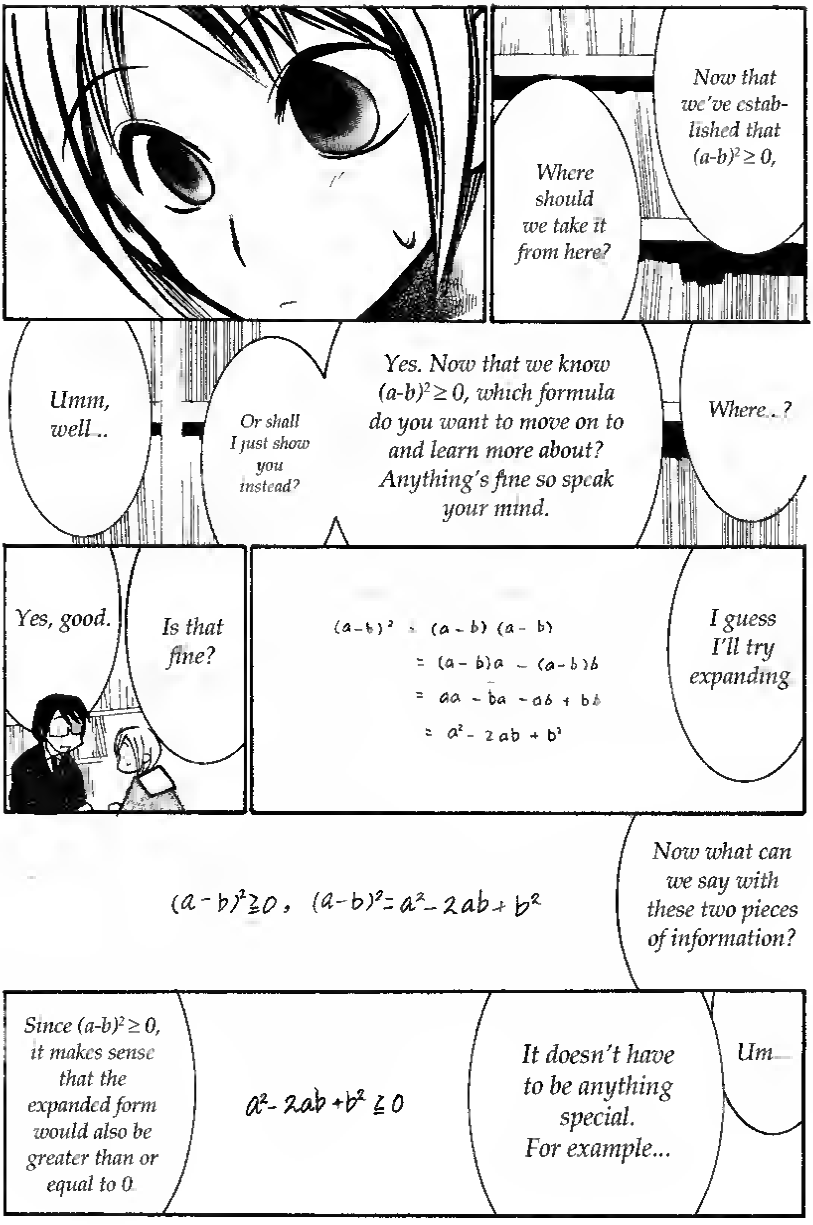

It's not that r^2 is greater than 0 but it's **at least as great** as zero. Otherwise, we'd be leaving the case when r is 0.
Almost
Umm... Since we squared it, I guess it would be bigger than 0? Am I right?
If r is a real number, then what can we say about r^2?
r^2
Yes, it doesn't matter whether r is a positive number, a negative number, or even 0. When r is multiplied by itself, it will be greater than or equal to 0, which is represented by the inequality $r^2 \geq 0$
This here, is an important aspect to keep in mind for real numbers. Don't forget that it's when r is 0 that we need the equal sign for
Umm...
Right, so r is at least as great as 0
$r^2 \geq 0$
Oh right. If r was 0, then r^2 would be 0 too.
Yes, so no matter which real number r is, it'll satisfy this inequality below.
Under-stand?
An inequality which is true for all values of the variable like what we have here is called an unconditional inquality
Since it works for any number we pick, in that aspect, it's quite similar to an identity, though there are other minor differences
Exactly.
Umm... This all seems like really obvious stuff...
Oh, okay
That's a good thing
The simple and obvious make great starting points

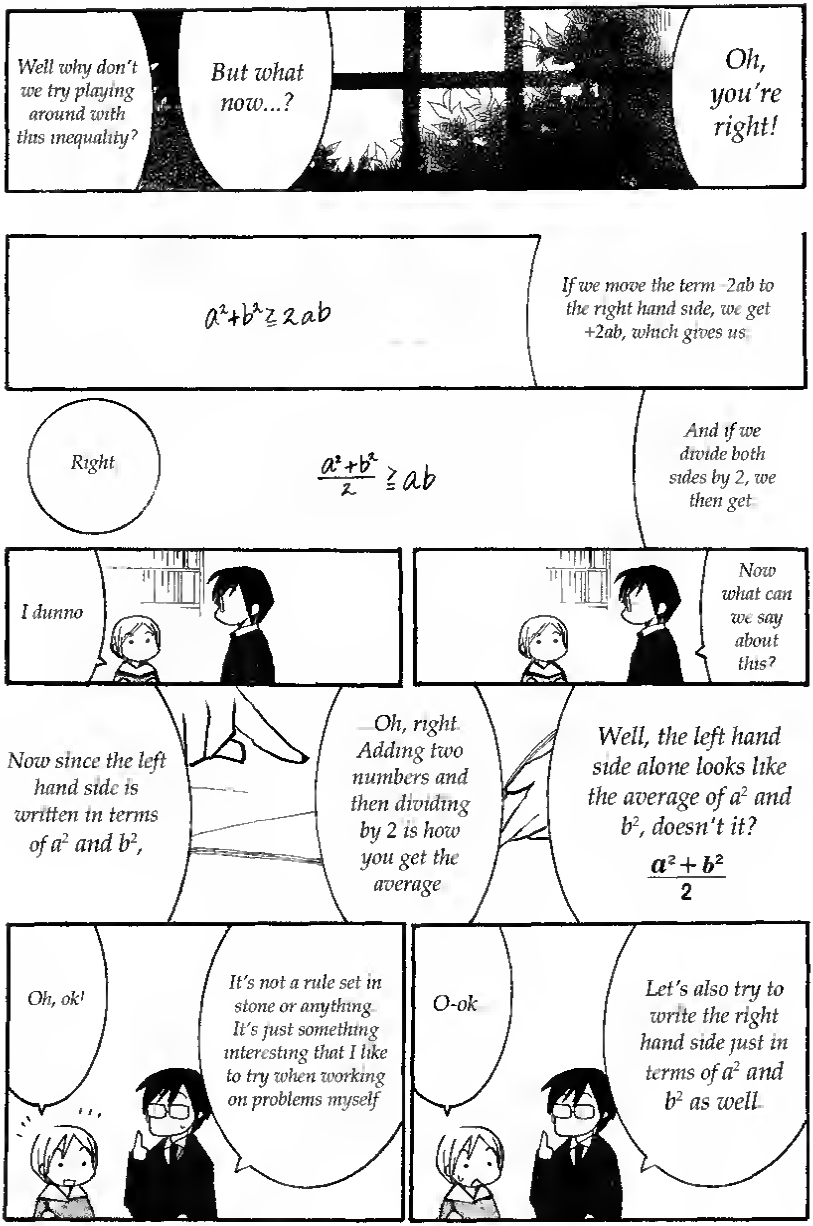
Now moving on, let's look at this example. If a and b are both real numbers, then we should be able to make this inequality,
$(a-b)^2 \geq 0$
Right?
Umm... Right. a-b would also be a real number so squaring that should be greater than or equal to 0 as well.
Ah-
One quick question before we move on any further
Why did we suddenly switch from using r as the variable to using a and b instead?
Ah, that's because r stands for "real number."
But you can have some other variable like w or x be the variable representing real numbers as well
Generally speaking, it's standard notation to use a, b, c to represent some constant number while using x, y, z for variables.
But don't be surprised when you see some other variable like n being used to represent real numbers. n is commonly used for integers or natural numbers, by the way
In any case, for our purposes, the letter we choose doesn't matter at all.
Yup!
Crystal clear!
Is everything clear?


Ah that's right. You told me before that you always get confused about the letters in math, didn't you?
Sorry for the random question all of a sudden

Hee hee, you remembered!
Thanks!


I-is that so?
I guess it is pretty odd, after all.
Yeah, I remembered because I thought it was a rather peculiar thing to get confused about.


Now that we've established that $(a-b)^2 \geq 0$,
Where should we take it from here?
Yes. Now that we know $(a-b)^2 \geq 0$, which formula do you want to move on to and learn more about? Anything's fine so speak your mind.
Umm, well ..
Or shall I just show you instead?
Where..?
Yes, good.
Is that fine?
$(a-b)^2 = (a-b)(a-b)$
$= (a-b)a - (a-b)b$
$= aa - ba - ab + bb$
$= a^2 - 2ab + b^2$
I guess I'll try expanding
Now what can we say with these two pieces of information?
$(a-b)^2 \geq 0$, $(a-b)^2 = a^2 - 2ab + b^2$
Since $(a-b)^2 \geq 0$, it makes sense that the expanded form would also be greater than or equal to 0
$a^2 - 2ab + b^2 \geq 0$
It doesn't have to be anything special. For example...
Um


Well why don't we try playing around with this inequality?
But what now...?
Oh, you're right!
$a^2+b^2 \geqq 2ab$
If we move the term -2ab to the right hand side, we get +2ab, which gives us
Right
$\frac{a^2+b^2}{2} \geqq ab$
And if we divide both sides by 2, we then get
I dunno
Now what can we say about this?
Now since the left hand side is written in terms of a^2 and b^2,
Oh, right Adding two numbers and then dividing by 2 is how you get the average
Well, the left hand side alone looks like the average of a^2 and b^2, doesn't it?
$\frac{a^2+b^2}{2}$
Oh, ok!
It's not a rule set in stone or anything It's just something interesting that I like to try when working on problems myself
O-ok
Let's also try to write the right hand side just in terms of a^2 and b^2 as well

$ab = \sqrt{a^2 b^2}$ (?)

Does this equation hold true for all real numbers a and b?

Now there's something we have to be careful about this next step.

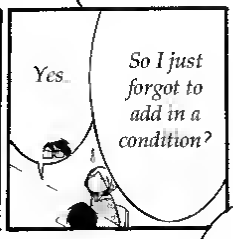
This is only true if ab is greater than or equal to 0. If ab were negative, then the root of the square would not return back a negative number. That's why we need to write a condition for this.

So yes, it does hold true.

Umm... The root of a square number is... the original number itself.

The left hand side is $ab = 2 \times -2 = -4$
While the right hand side is
$\sqrt{a^2 b^2} = \sqrt{2^2 \cdot (-2)^2} = \sqrt{16} = 4$

You can see this clearly if we try letting $a = 2$ and $b = -2$.



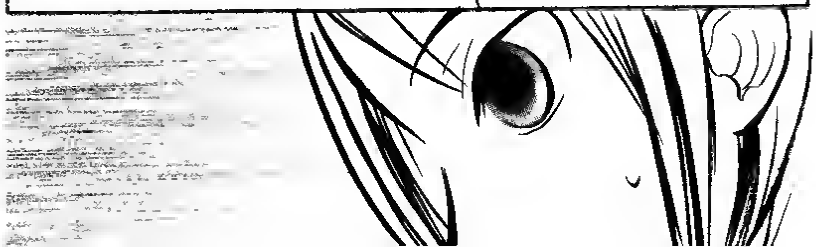
$ab = \sqrt{a^2 b^2}$ where $ab \geq 0$

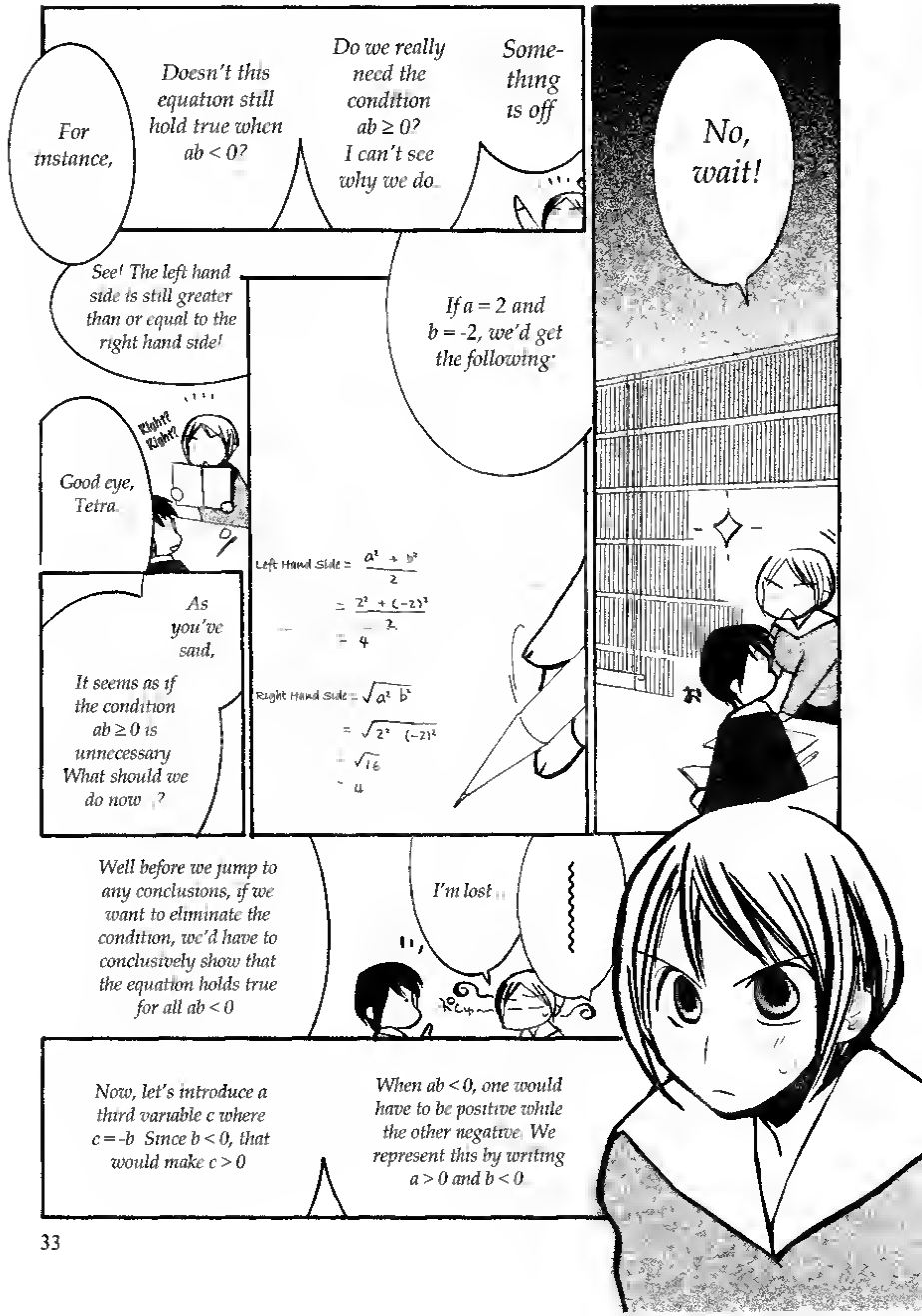
That's why we'll add in the condition that ab must be greater than or equal to 0.

Oh, you're right.

$\frac{a^2 + b^2}{2} \geq \sqrt{a^2 b^2}$ where $ab \geq 0$

Now, if we put this back into the original equation, we get:




For instance,
Doesn't this equation still hold true when ab < 0?
Do we really need the condition ab ≥ 0? I can't see why we do.
Something is off
No, wait!
See! The left hand side is still greater than or equal to the right hand side!
If a = 2 and b = -2, we'd get the following.
Right? Right?
Good eye, Tetra.
Left Hand Side = (a² + b²)/2
= (2² + (-2)²)/2
= 4
Right Hand Side = √(a² b²)
= √(2² (-2)²)
= √16
= 4
As you've said,
It seems as if the condition ab ≥ 0 is unnecessary. What should we do now...?
Well before we jump to any conclusions, if we want to eliminate the condition, we'd have to conclusively show that the equation holds true for all ab < 0
I'm lost...
Now, let's introduce a third variable c where c = -b. Since b < 0, that would make c > 0
When ab < 0, one would have to be positive while the other negative. We represent this by writing a > 0 and b < 0

$$\frac{a^2+c^2}{2} \geq ac$$

Now if $\frac{a^2+b^2}{2} \geq ab$ were to hold true for any real number a and b, the same would apply for a and c in this next inequality.

Now let's separately examine each side of this inequality

Left Hand Side $= \frac{a^2+c^2}{2}$

$= \frac{a^2+(-b)^2}{2}$ since $c = -b$

$= \frac{a^2+b^2}{2}$

We thus obtain this.

$\frac{a^2+b^2}{2} \geq \sqrt{a^2b^2}$ where $a>0$ and $b<0$

Now, in getting to this, we initially said that $a > 0$ and $b < 0$. But you can easily see how we'd get the same exact inequality even if we originally said $a < 0$ and $b > 0$. So in short, we've now shown that the inequality holds when either a and b are positive or negative real numbers. Thus, we write

$\frac{a^2+b^2}{2} \geq \sqrt{a^2b^2}$ where a and b are any* real number

*Japanese term used is "nmi"

Right Hand Side $= ac$

$= \sqrt{a^2c^2}$ since $ac > 0$

$= \sqrt{a^2(-b)^2}$ since $c = -b$

$= \sqrt{a^2b^2}$

Oh, I get it now!

But one more thing

tl note: unlike the english term "any real number", the japanese equivalent is less obvious to understand, hence Tetra's question

What does "nini" exactly mean?

All that means is that it won't matter which number we specifically pick. It can be anything.

In English, the words "any" or "for all..." are commonly used to express that same idea.

Oh, alright. So it doesn't matter which specific real number we choose because it'll all work, right?

Right. Now then, let's substitute a and b with a different set of variables. For a^2, we'll use x, while for b^2, we'll use y. Before we can just make the switch, it's neccessary to write a line like this.

Let $x=a^2$, $y=b^2$

$\frac{x+y}{2} \geq \sqrt{xy}$ where $x \geq 0$ and $y \geq 0$

Since a and b are real numbers, taking the square would make them greater than or equal to 0 as we previously discussed. Thus, our new condition becomes $x \geq 0$ and $y \geq 0$.

Now, have we ever seen something similar to this?

In the left hand side, we have $\frac{x+y}{2}$

We're adding two numbers and dividing by two, which is how we calculate the arithmetic mean of two numbers.

In the right hand side, we have $\sqrt{xy}$

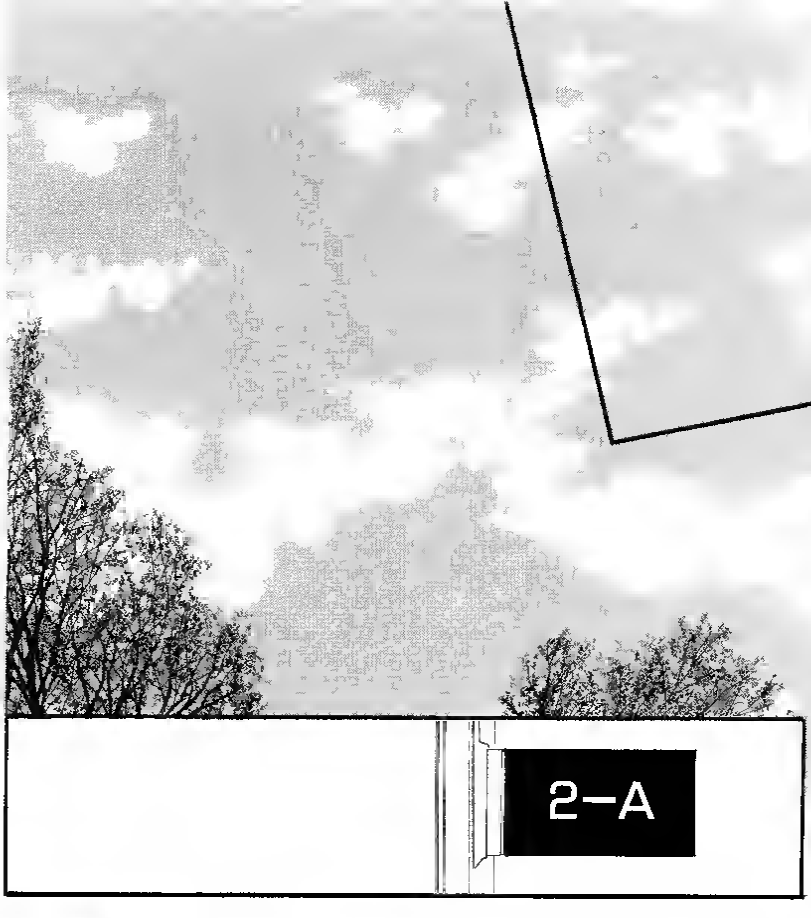
We're multiplying two numbers and taking the square root, which is how we calculate the geometric mean of two numbers. What we've proven here with our inequality is that the arithmetic mean of two numbers is greater than or equal to their geometric mean.

Good.

Ah, I know!

It's the relationship between arithmetic and geometric means, right?

See how we arrived at an entirely new formula just from the simple statement that $r^2 \geq 0$?


2-A

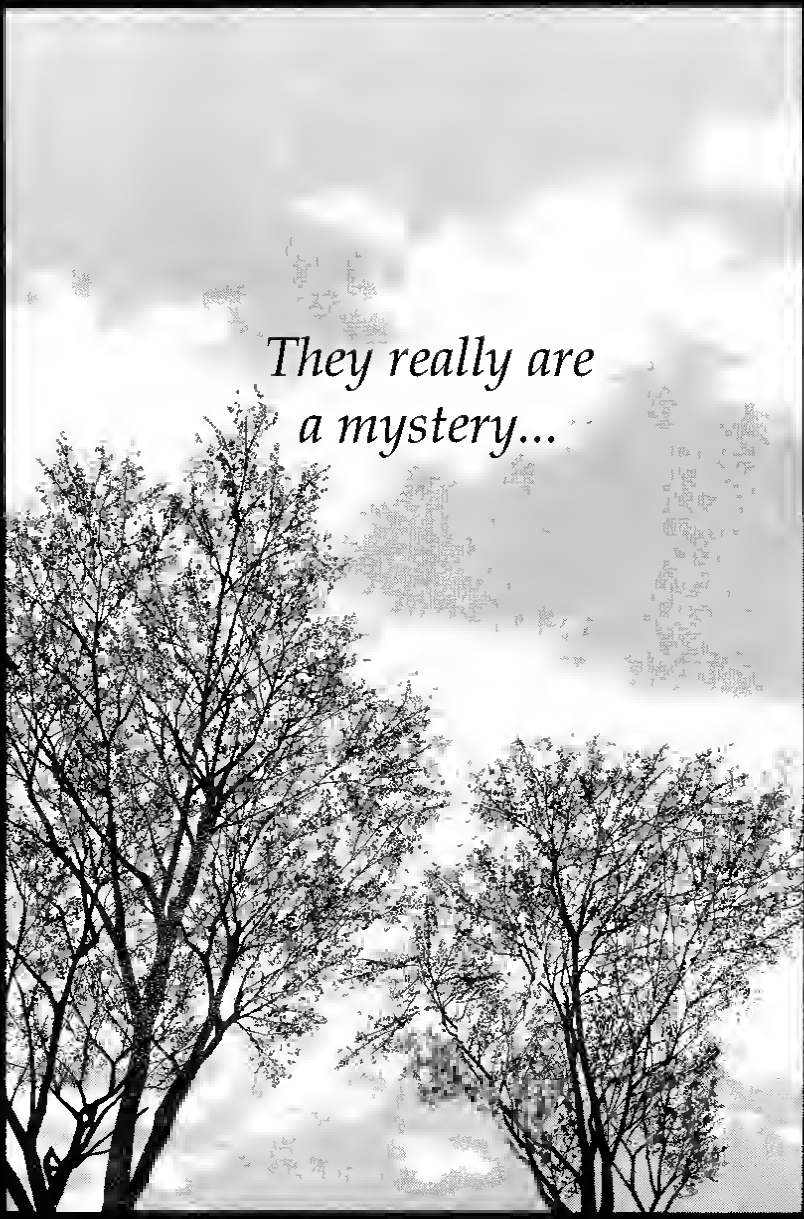
Hey!
But for that alone...
I want to believe.
Hadn't be done on a whim.
.....
Just the usual talk about math,
As if nothing had happened...
Mika
Tetra
Me
If that too had been just another whim of hers
I wonder,

There's someone here for you.
Some girl that looks like she could be your little sister.
Ah-
S-
Sorry about coming unexpectedly,
To your classroom and all...

But what's with the little sister comment? Gimme a break.
I think it's fine.
Why don't you try calling him "brother?"
I bet it'd make him happy.
Am I right?

I almost forgot.
Ah, that's right.
Ohh, maybe I should then~♬
uhh
Milka,
If it's alright with you,
I'd like to give these snacks I have lying around at home
I'm trying to diet right now,
So having them at my house is just tempting me to eat..
Sure, I'll take it.

Despite something like that in the not-too-recent past,
What is it,
About girls...
They're already this close together...
Girls...


They really are
a mystery...

T I P S

[*Relationship between Arithmetic and Geometric Means*]

A theorem that states the arithmetic mean between two numbers will be at least as great as their geometric mean. This is represented by the equation. $\frac{a+b}{2} \geq \sqrt{ab}$

When a = b, the two sides are equal to each other.

数学ガール

Mathematical Girls

Ah!
Huh?
The mind's process when solving problems...
Is quite similar to using inequalities to estimate the size of expressions.
Senpai!

Oh hi, Tetra.
Chapter 10

Planetarium
Coin Lockers
I recently received some free tickets so I thought I might as well try coming
Oh, really?
Same here.
"It seems like today's the last showing of this month's exhibit.
Stars should...

Technically be finite in number,
But there's so many that you just want to write it off as infinite instead.
Speaking of which, let's think about infinite series.
.....
She casually changes any topics she doesn't feel like talking about.
For the infinite geometric series $1+x+x^2+x^3+\ldots$,
Just like this,
$1+x+x^2+x^3+\cdots+x^n=\frac{1-x^{n+1}}{1-x}$
But I've long become used to this.
The partial sum of the first n terms is given by the above formula.

Now if the absolute value of x is less than 1, written as $|x|<1$, then when n approaches infinity, x^{n+1} will approach 0. Thus, the formula is simplified as shown below.

Despite that...

$$1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$$

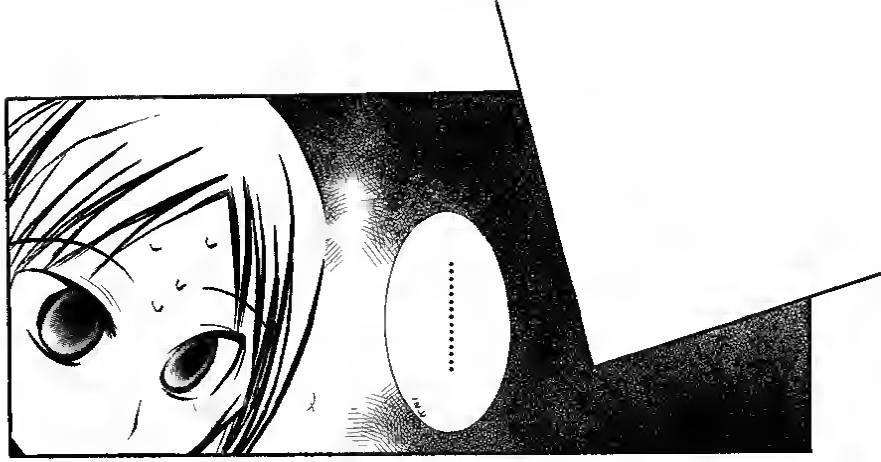
The sum of the infinite geometric series is given by:

$$1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$$

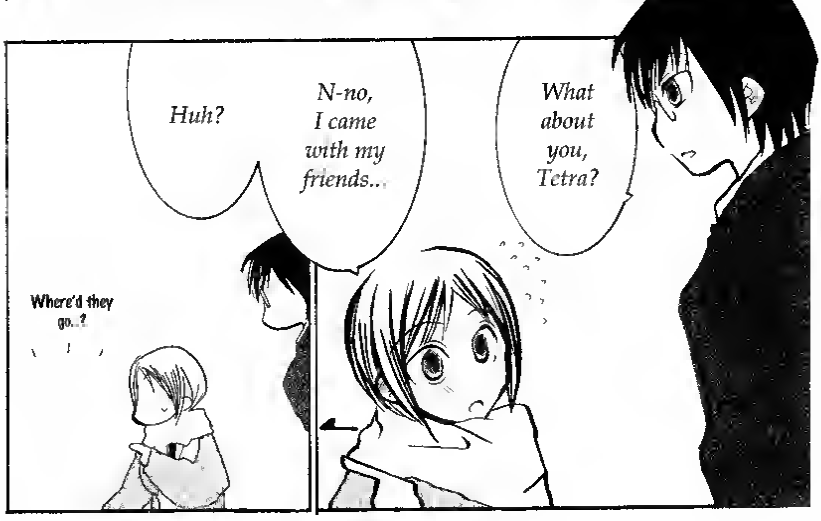
where the 0th term is 1 and $|x| < 1$

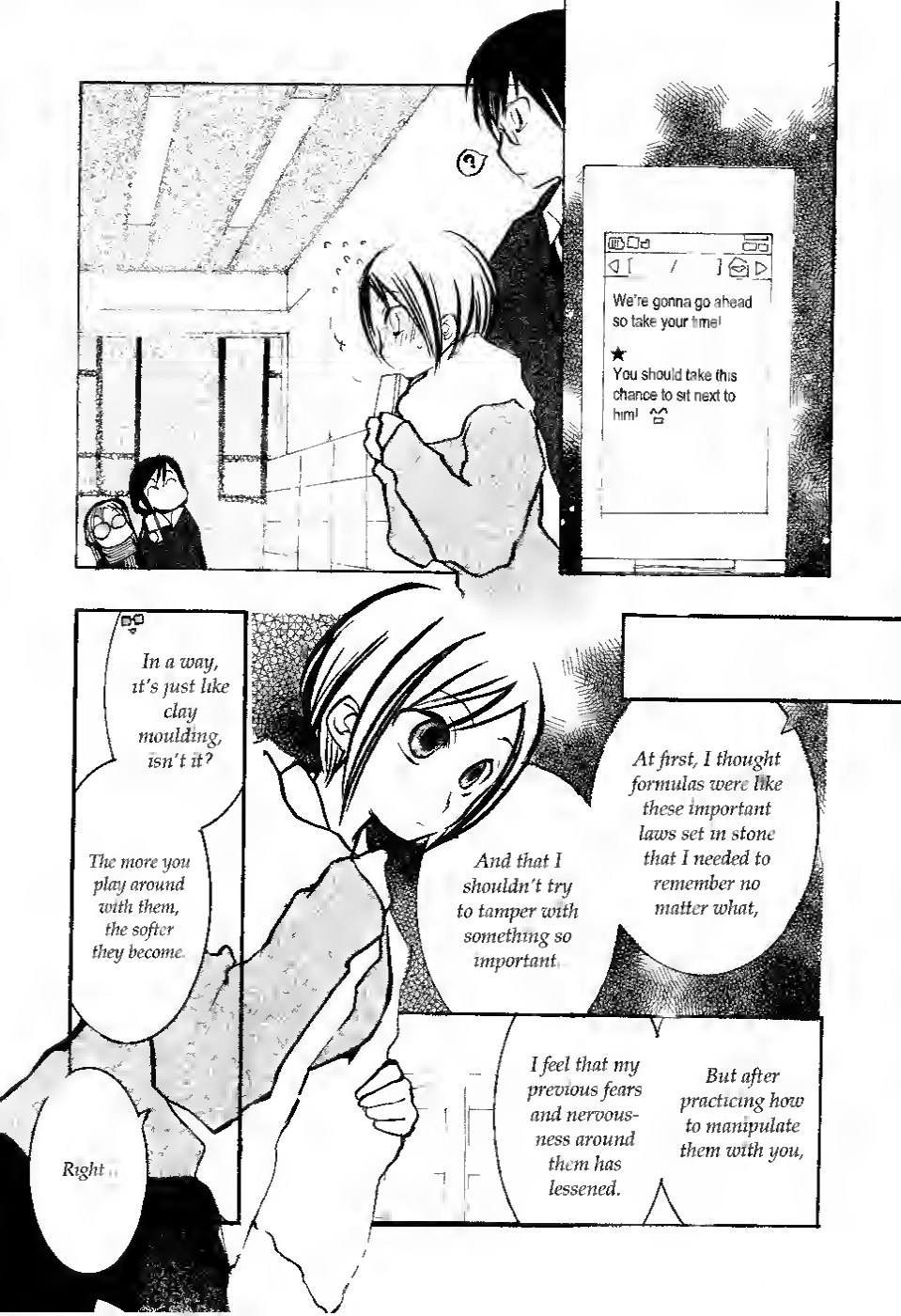
We have an infinite series
That we can't fully write out because there's an infinite number of terms
In the left hand side
To express the sum of an infinite series,
In such a compact form...
Whereas on the right hand side,
We have one finite number

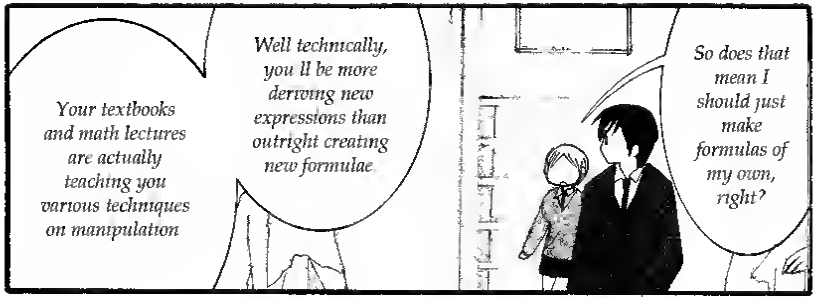
Isn't that just lovely,
Well...?

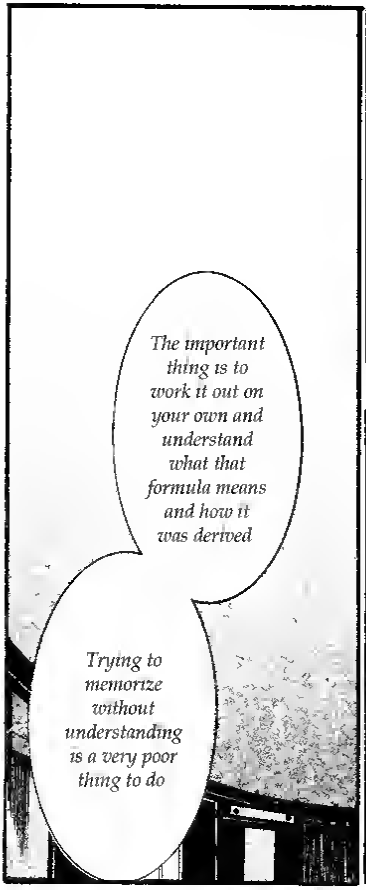

.....

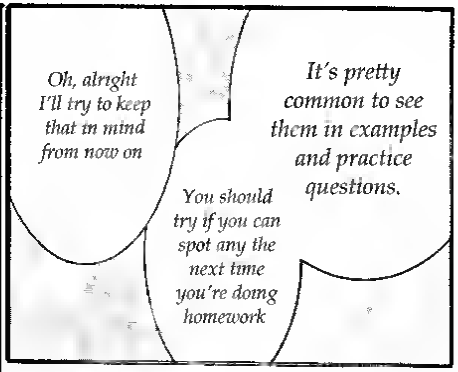
Ah,
Um...
Are
Are
y y y-y
y-y-y-you
H here all
b b-by
Y-y-y-
yourself
or...?
Yeah, I
came by
mself

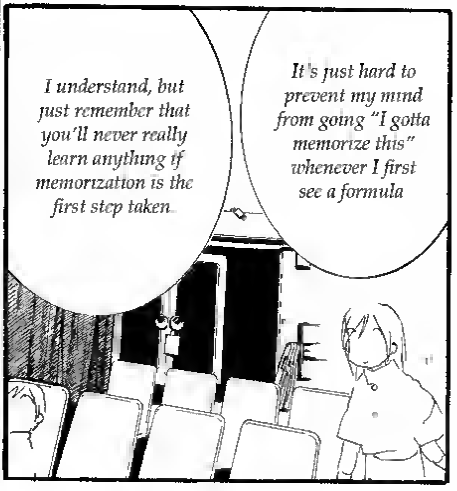

What
about
you,
Tetra?
N-no,
I came
with my
friends...
Huh?
Where'd they
go...?


We're gonna go ahead so take your time!
★
You should take this chance to sit next to him! ^^
?
In a way, it's just like clay moulding, isn't it?
The more you play around with them, the softer they become.
Right...
And that I shouldn't try to tamper with something so important.
At first, I thought formulas were like these important laws set in stone that I needed to remember no matter what,
I feel that my previous fears and nervousness around them has lessened.
But after practicing how to manipulate them with you,


Your textbooks and math lectures are actually teaching you various techniques on manipulation
Well technically, you'll be more deriving new expressions than outright creating new formulae
So does that mean I should just make formulas of my own, right?


The important thing is to work it out on your own and understand what that formula means and how it was derived
Trying to memorize without understanding is a very poor thing to do


Oh, alright I'll try to keep that in mind from now on
You should try if you can spot any the next time you're doing homework
It's pretty common to see them in examples and practice questions.


I understand, but just remember that you'll never really learn anything if memorization is the first step taken
It's just hard to prevent my mind from going "I gotta memorize this" whenever I first see a formula

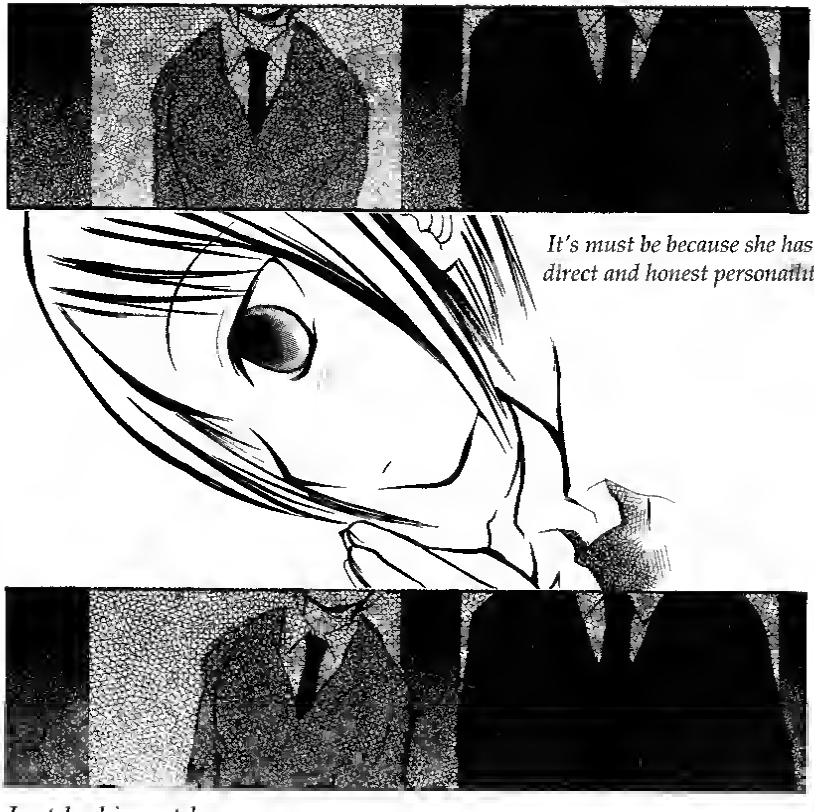
I never would have imagined,
That I'd become like this
Looks like our conversation changed to math again
Back at this time last year...
Ha, I guess it did

I wasn't even able to start a conversation,
Even if I wanted to

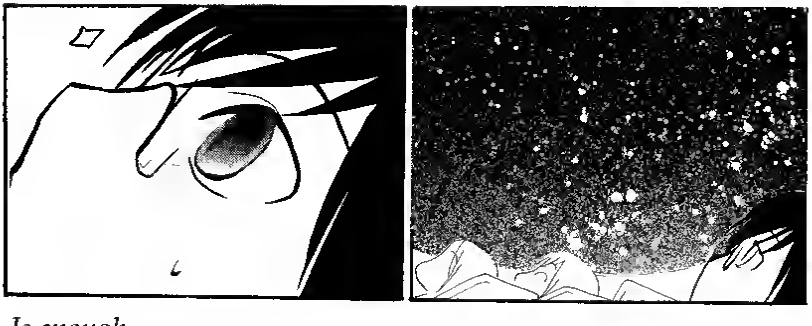
A-all I meant by that was that I couldn't talk about math even if I wanted to in the past!
Oh...
Ah-
Huh...?
YES!
So that's what you meant...
O-oh look, the show's about to start! Haha...
EXIT
I, uhh...
....Oh.

That surprised me
for a moment...

Wow...
She's expressive, isn't she
Whenever she's happy, troubled, or deep in thought,
Look right here!
Unique prime-factori-wha?
What does that mean...?
She lets it directly show on her face



Just looking at her,



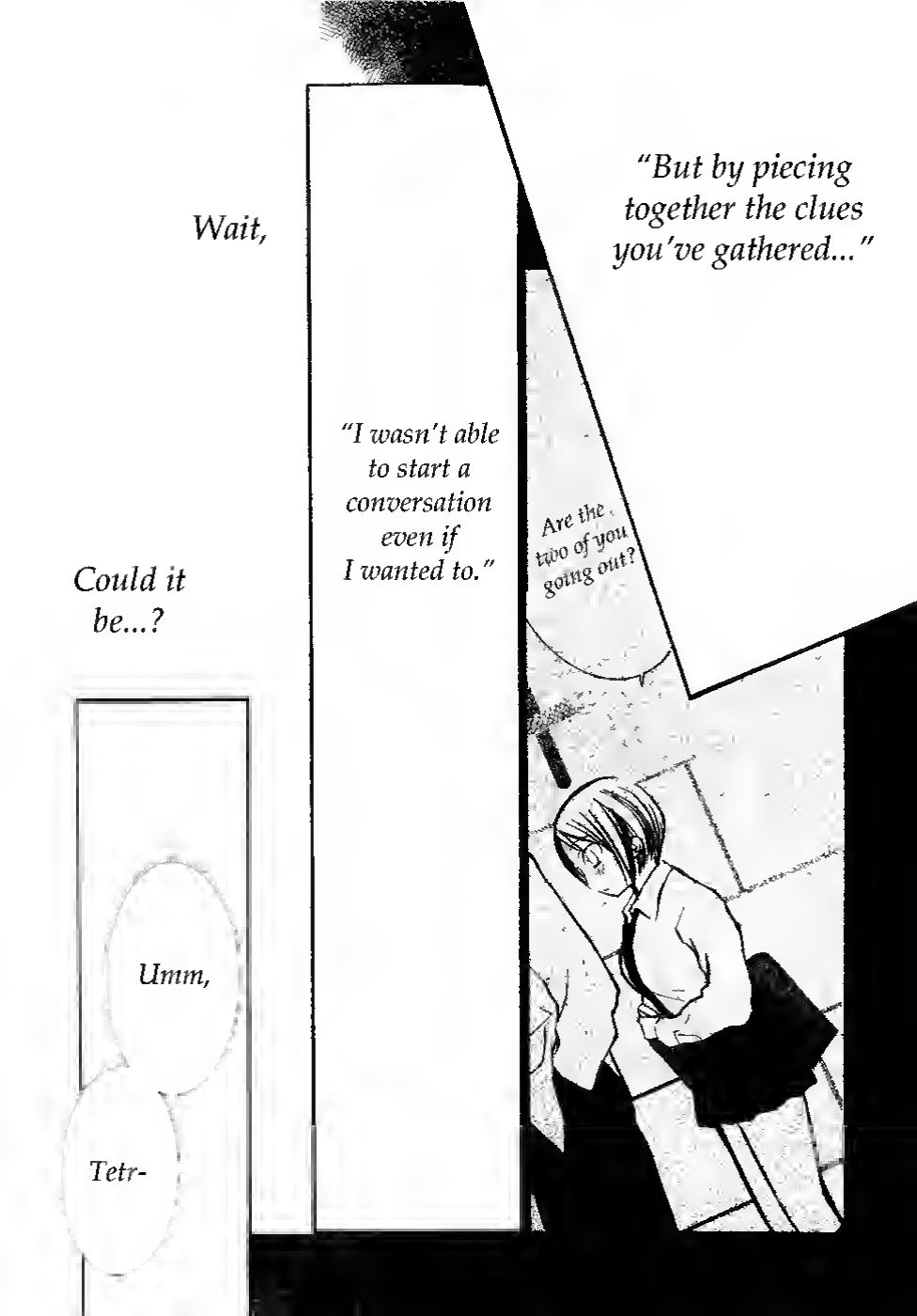
Is enough,

To make even me and my feelings,

Our talk last time was really interesting
I never would've thought such a simple inequality as $r^2 \geq 0$,
Would tell us so much
I think that using inequalities to evaluate the size of expressions
Is quite similar to the way your mind works when solving any problem

Be put at ease as well...

Is that the answer is greater than this but less than that...
Just having the inequality doesn't guarantee you to an answer.
The only thing you'll know for sure,
You won't be able to immediately understand everything
But by piecing together the clues you've gathered
You'll be able to reach closer and closer to the answer


Wait,
"But by piecing together the clues you've gathered..."
"I wasn't able to start a conversation even if I wanted to."
Are the two of you going out?
Could it be...?
Umm,
Tetr-

ぎゅ
……

Wake up!
Hey!
Tetra~

T I ρ S

[*Planetarium*]

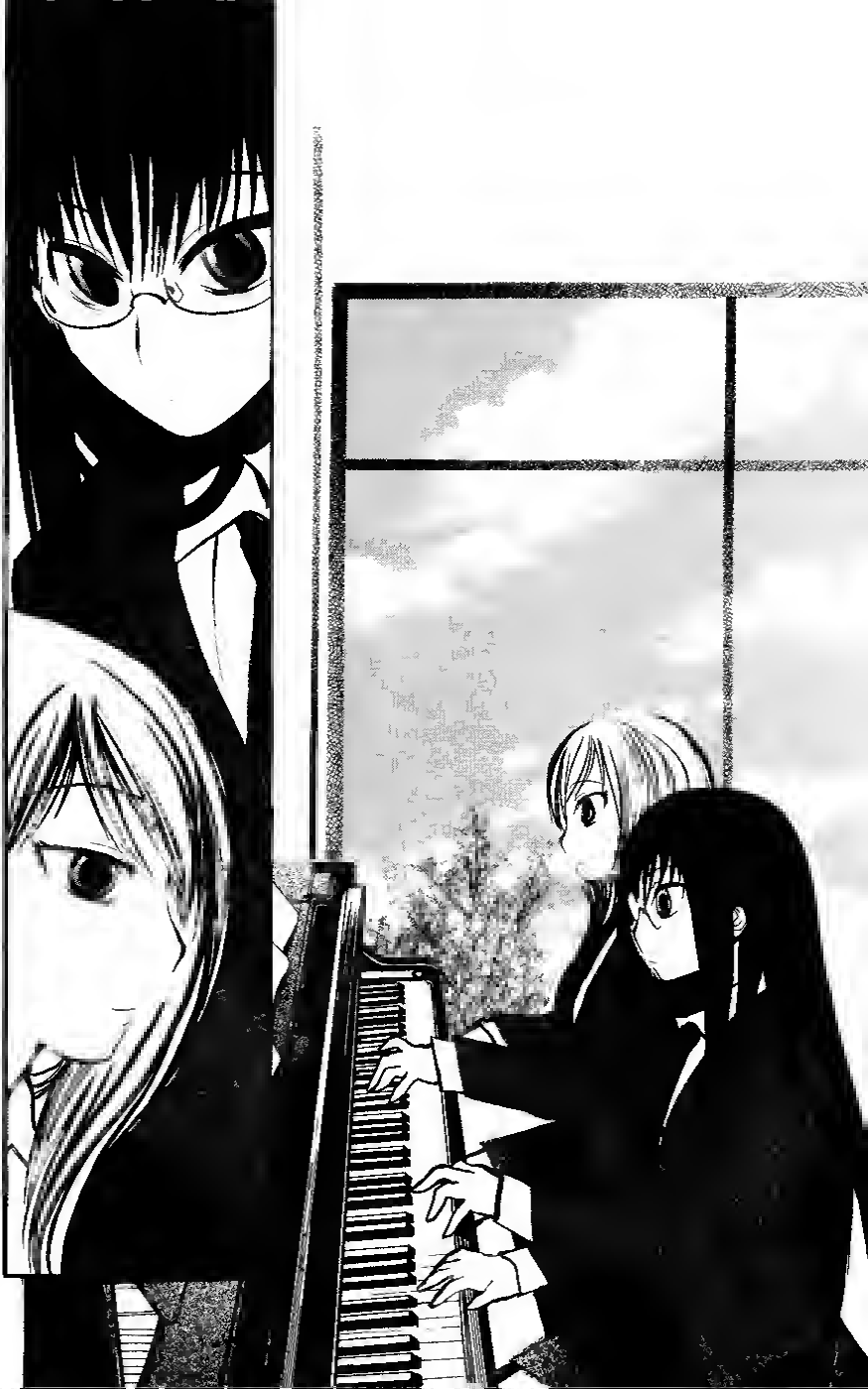
A theoter, or the focility housing it, thot projects the night sky ond its movements for educotionol ond entertoinment purposes. The etymology derives from the combinotion of "plonet" and "-arium," a suffix meoning ploce. Japon is ane of the leoding countries in the number of plonetoriums. At plonetoriums, people ore free to enjoy the night sky, or like Tetro, enjoy the night sky in their dreoms.

数学ガール

Mathematical Girls

Cold day, isn't it?
Morning, Milka.

Chapter 11

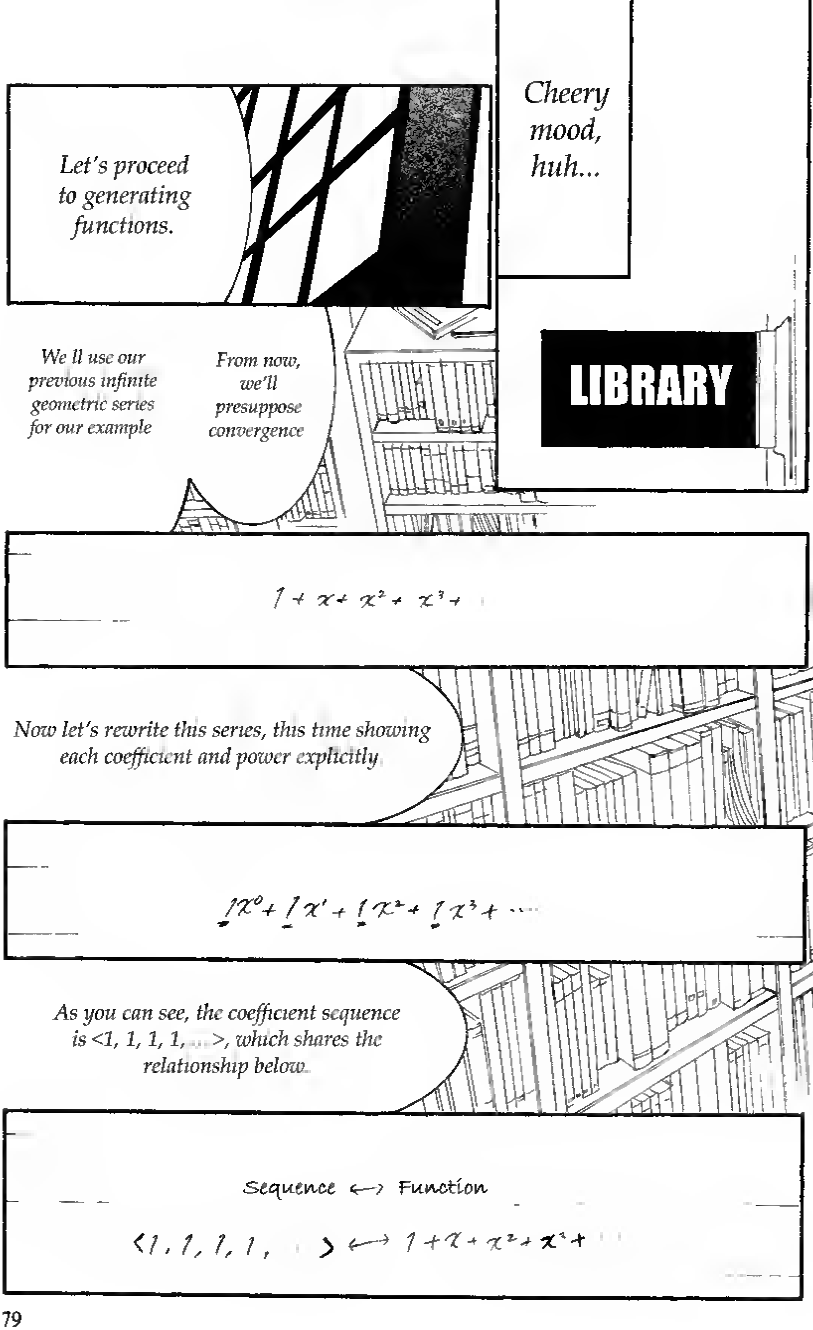


Did you like it?
It was only a gradual climbing of the musical scale and yet
It was...
Almost as if there were an infinite set of notes coming from a finite set of keys

As if repeating a certain phrase over and over,
The musical scale kept on climbing and climbing,
She'll hit the end soon though
All the way until it felt as if it had gone beyond the highest key in the piano...
But just then...
Huh? When did she switch to lower notes?
Like climbing an endless set of stairs
The music, too, continued to climb endlessly
Like a series diverging into the infinite, isn't it?
To have the finite diverge would seem like quite a contradiction though
Looking at her hands I see that she indeed returned to the left side of the piano
That's right.
By shifting to a different octave,
You switched to a higher octave, didn't you?
I can continue to climb the scale.
But to my ears, it still sounded as if the notes were climbing.

Creating an illusion to the human ear that the notes are climbing endlessly.
And the higher notes are played softer in comparison with the lower notes,
-Infinite rising of the Musical Scale-
Don't go revealing all the secrets now.
Of course, doing it with just two hands has its limits so we did it together.
A simple musical scale would be boring, no?
That's why I thought about upping the tempo to keep things interesting for the listener but by losing the simplicity, it also loses its mysterious charm.
Ahh-
Eiei.
It's hard coming up with a piece, you know.

What kind of song is that?
Next time, let's try a mobius strip harmony.
Sea Anemone
It's a good thing that Milka's hands are so nimble


Let's proceed to generating functions.
Cheery mood, huh...
LIBRARY
We'll use our previous infinite geometric series for our example
From now, we'll presuppose convergence
$1 + x + x^2 + x^3 + \cdots$
Now let's rewrite this series, this time showing each coefficient and power explicitly
$\frac{1}{0!}x^0 + \frac{1}{1!}x^1 + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots$
As you can see, the coefficient sequence is <1, 1, 1, 1, ...>, which shares the relationship below
Sequence ↔ Function
$\langle 1, 1, 1, 1, \ldots \rangle \leftrightarrow 1 + x + x^2 + x^3 + \cdots$

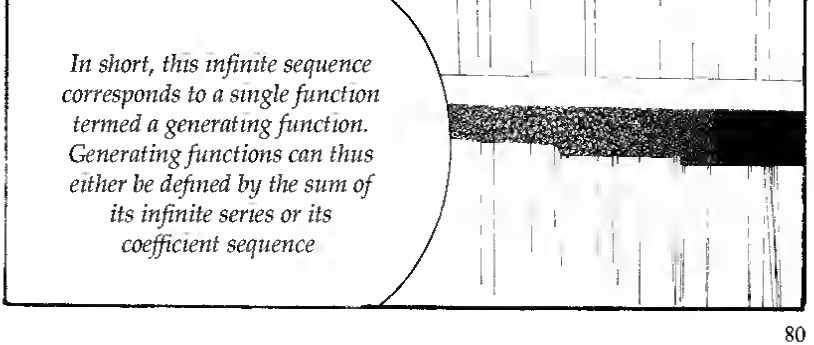
In other words, <1, 1, 1, 1, ... > would be the sequence equivalent of $1+x+x^2+x^3+\cdots$, *which converges to the sum* $\frac{1}{1-x}$. *Thus we can write the following below:*

Sequence $\leftrightarrow$ Function

$\langle 1, 1, 1, 1, \ldots \rangle \leftrightarrow \frac{1}{1-x}$

Sequence $\hookrightarrow$ Function

$\langle a_0, a_1, a_2, a_3, \ldots \rangle \leftrightarrow a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$


In short, this infinite sequence corresponds to a single function termed a generating function. Generating functions can thus either be defined by the sum of its infinite series or its coefficient sequence

.....
Right now,
We came up with a generating function corresponding to a sequence
Thus for any sequence, we can find a corresponding closed-form generating function
Now,
Here's a thought. .

I'm thinking of traversing across two lands right now
In order to catch a sequence
To do so directly is difficult, however.
But I can alternatively cross over from the land of series to the land of generating functions.
There, I make my way through various functions,
Before returning to the land of sequences
That would finally allow me to catch my sequence, wouldn't it?
...What do you mean?

Sequence ⟷ Generating Function
⟨a₀, a₁, a₂, ⋯⟩ ⟷ a₀+a₁x+a₂x²+⋯
Perhaps my metaphor was too abstract
By travelling between the two lands, I mean that we can use generating functions to find a general term of a sequence
Let me write the itinerary for our travels
Given a sequence, the corresponding generating function is first found. Then, by manipulating that function, a closed-form expression is found, which will then express the general term of the original sequence
In other words, the general term of a sequence is found by temporarily utilizing generating functions
For instance, take the Fibonacci sequen-
"Itinerary from a sequence to its general term expression"
Sequence → Generating Function
↓
General term of Sequence ← Closed-Form Expression of Generating Function

You surprised me for a second there.
I didn't know you two were both here

Tetra,
Do you know about the Fibonacci sequence?
Yup!
〈0, 1, 1, 2, 3, 5, 8…〉
It's where each term is the sum of the previous two terms,
Correct
You taught me that just the other day
Right!?
Though some start with 1 as the first term…

0, 1, 0+1=1, 1+1=2, 1+2=3, 2+3=5, 3+5=8

Given that $F_0=0$ and $F_1=1$, the Fibonacci sequence is defined as $F_n=F_{n-2}+F_{n-1}$

Let's start off with 0 in our case.

Fibonacci Sequence Definition (Recurrence Relation)

$$F_n=\begin{cases}0 & (n=0)\\ 1 & (n=1)\\ F_{n-2}+F_{n-1} & (n\geq 2)\end{cases}$$

In short, they can be summarized by the recurrence relation shown here

It's neccessary to calculate it successively, as in going from $F_0, F_1, F_2, \ldots$

Thus, to determine each term,

From this, it's clear that every Fibonacci number after 0 and 1 require two previous terms to be defined.

That is what I mean when I say I can't directly catch the sequence.

That is to say, it's not defined strictly in terms of n alone and must rely on F_{n-2} and F_{n-1}

Hence, F_n is not in a closed-form.

Now, if I were to ask, "What is the 100th term in the Fibonacci sequence,"
Then...

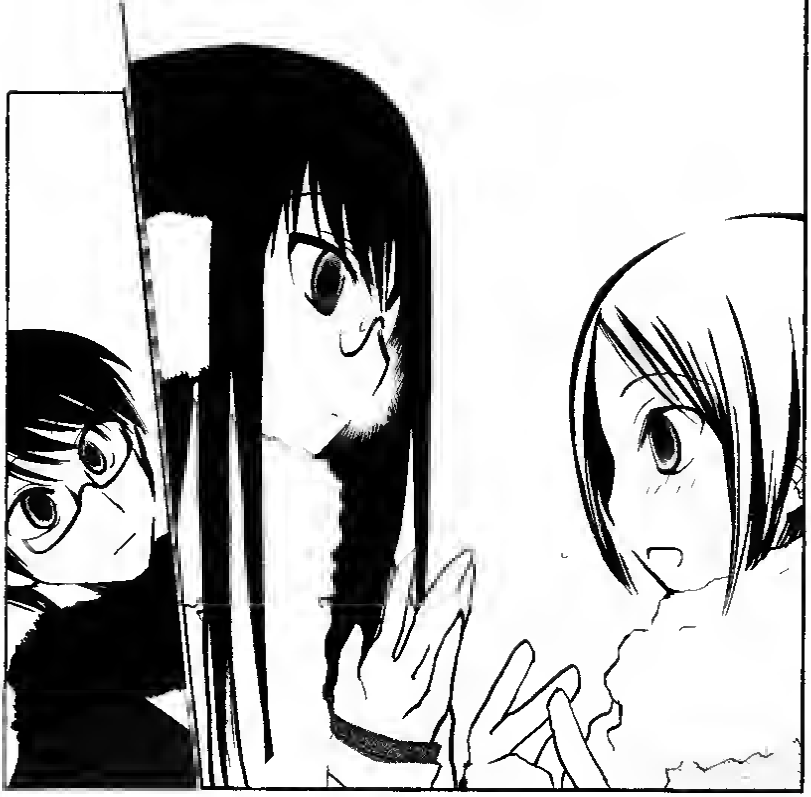
We'd have no choice but to painstakingly calculate each term in the sequence until reaching F_{98} and F_{99}, which we then add to find our answer.
Thus, given the sequence's recurrence relation, to find the nth term, we need to do n-1 number of calculations.
Well, that's obvious, isn't it?
True—
But what I want,
Is to find a closed-form expression for Fn, that is to say, I want to define Fn purely in terms of n and standard mathematical operations such as addition or division
By doing so, we'll finally be able to catch the Fibonacci sequence

Question:
Find the closed-form expression of the general term F_n in the Fibonacci sequence
It's closing hours.

It's the Fibonacci sign!
What're you doing?
Ah-
1,
1,
2,
3,
5,
...Huh?
See here.
-Tetra-

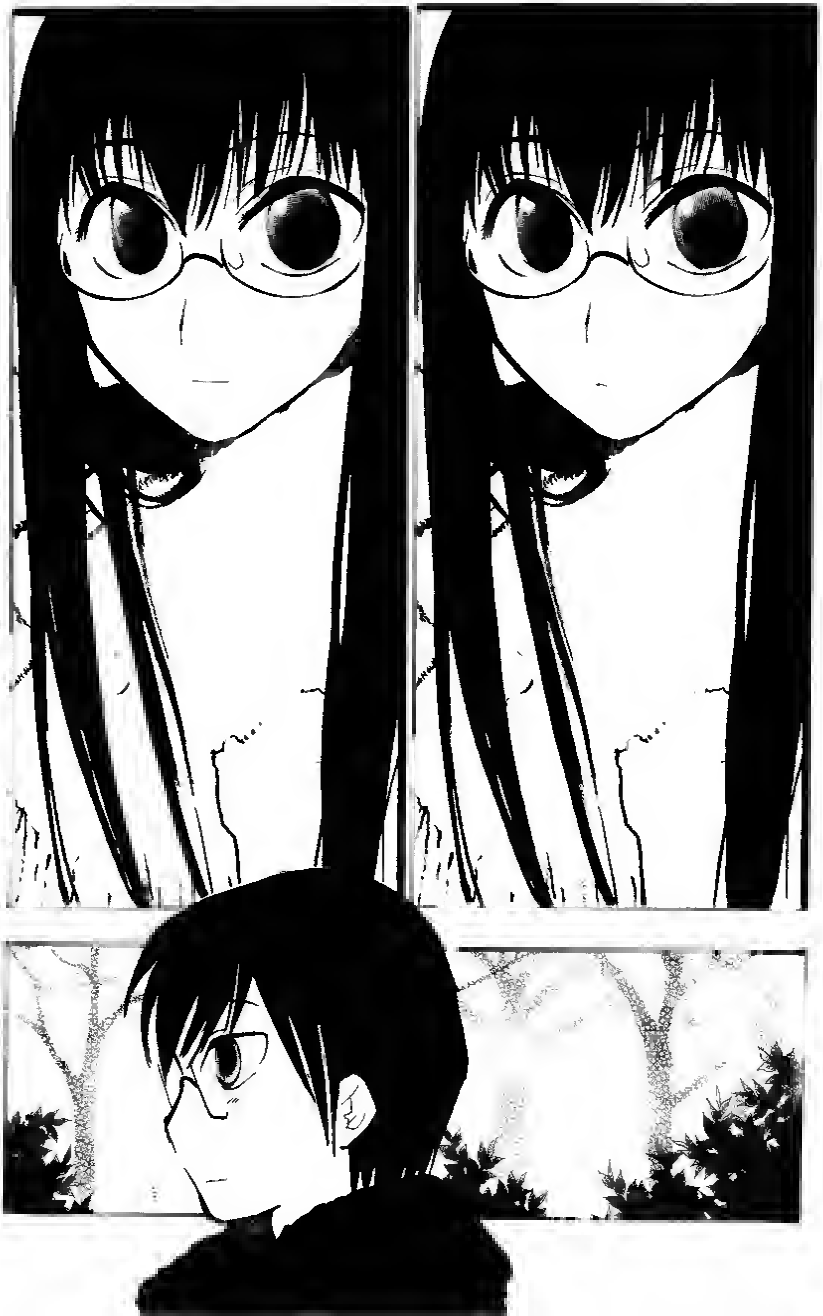
The fingers
1
1
2
3
5
Form the numbers in the sequence.
That's why it's the Fibonacci sign!
1,
1,
2,
3,
Milka,
About the problem before...
It'll be like our secret sign between us math nerds!
Stomp!
Don't just ignore me like that!

5
Tetra,
What do you do after 5?
M-milka ...?
Should I use both hands for 8?
No you stop at 5
If you sign a 1, 1, 2, 3,
The other person responds with a 5.



Me!?
You too, Senpai!
Umm..
Come on!

Bye then!
5
4
3
2
1
<3
Her cheerfulness really is contagious, isn't it?



really
Nope,
Foul mood...
I think...?

T I P S

[*Generating Function*]

A formal power series whose coefficients form a sequence. Generating functions turn an infinite sequence into a single function and is thus an important concept in studying sequences.

数学ガール

Mathematical Girls

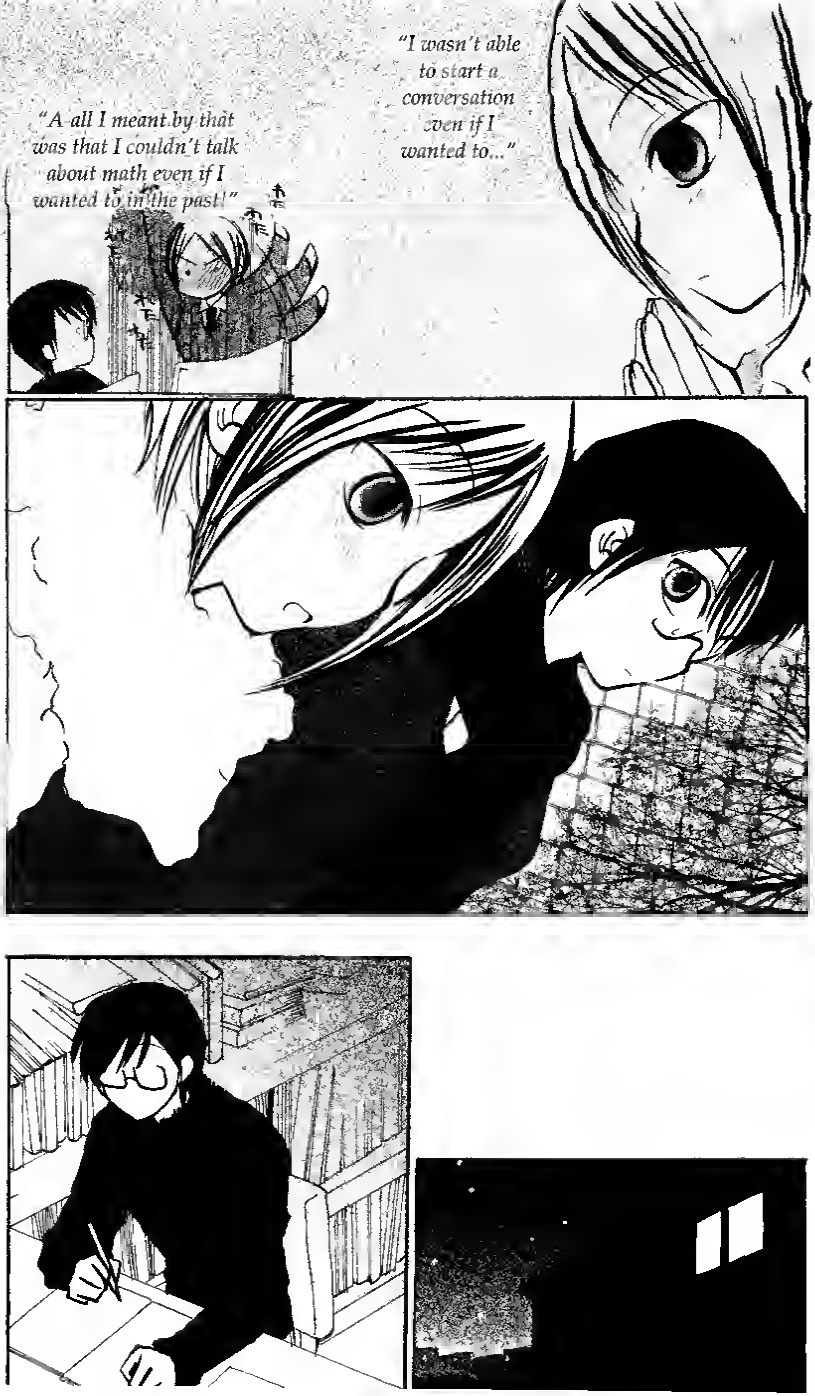
When I,
Stand in front of the science and mathematics section at the bookstore,

I realize
I still don't
understand
the contents of
most of these
books here.
Senp...

Chapter 12

Umm... About the other day...
Huh?
About the planetarium, I mean...
I'm sorry!

Hmm,
Now that I think about it,
I just kept forgetting to mention it the past few times I saw you...
O-oh...
I guess this is the first time we've been alone since that time...


"I wasn't able to start a conversation even if I wanted to..."
"A all I meant by that was that I couldn't talk about math even if I wanted to in the past!"

Question: Find the closed-form expression of the general term F_n of the Fibonacci sequence

First, convert the Fibonacci sequence to the generating function $F(x)$.

Sequence $\leftrightarrow$ Generating Function

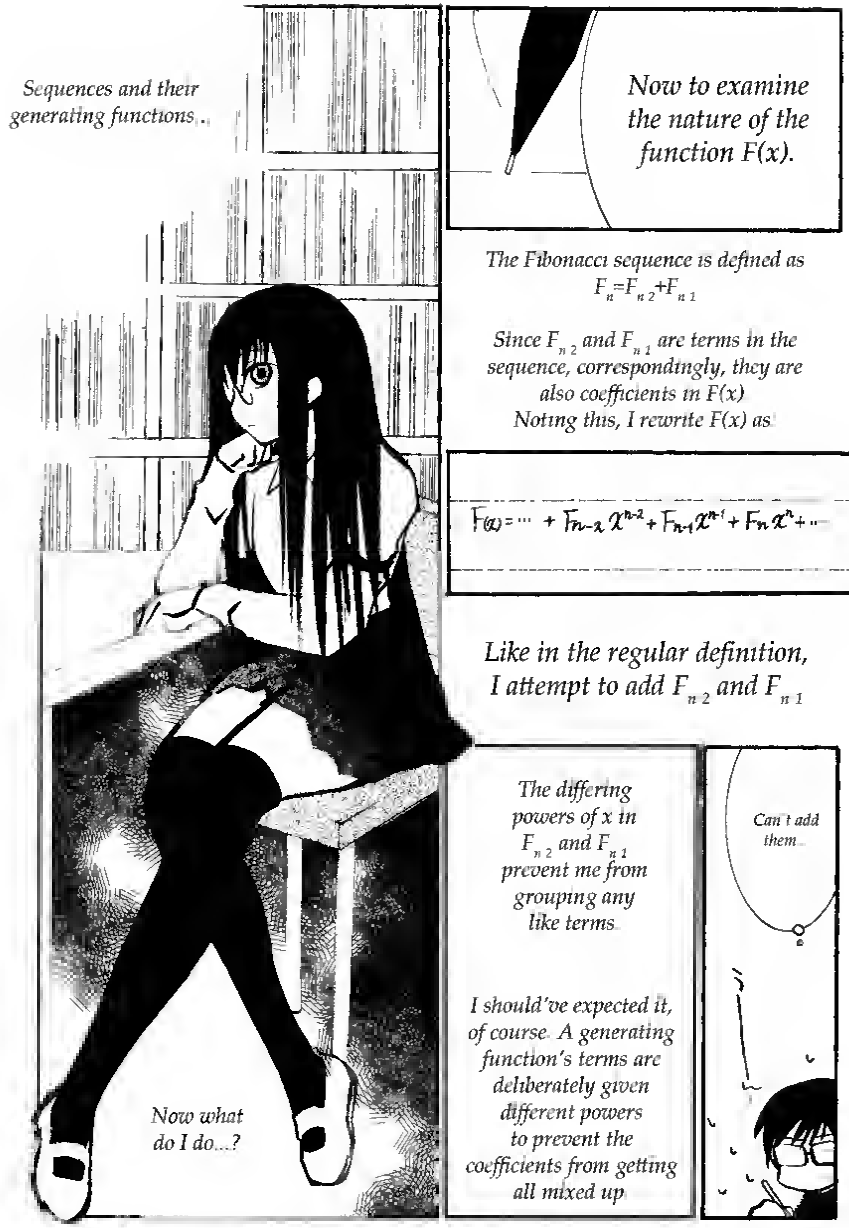
$\langle F_0, F_1, F_2, F_3, \ldots \rangle \leftrightarrow F(x)$

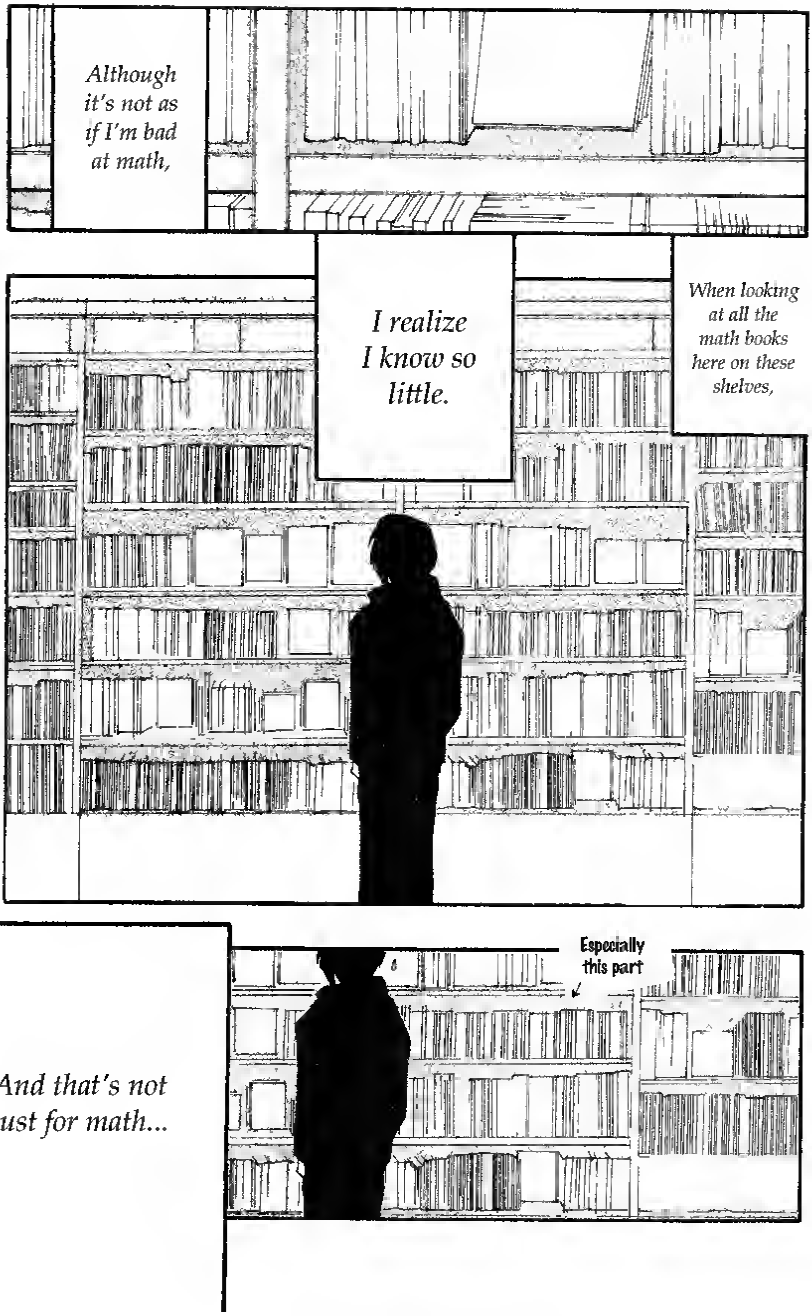
The coefficients of $F(x)$ will form the Fibonacci sequence

I cross into the land of functions.

And so,

$$\begin{aligned} F(x) &= F_0x^0 + F_1x^1 + F_2x^2 + F_3x^3 + F_4x^4 + \cdots \\ &= 0x^0 + 1x^1 + 1x^2 + 2x^3 + 3x^4 + \cdots \\ &= x + x^2 + 2x^3 + 3x^4 + \cdots \end{aligned}$$


Sequences and their generating functions...
Now what do I do...?
Now to examine the nature of the function F(x).
The Fibonacci sequence is defined as $F_n=F_{n-2}+F_{n-1}$
Since F_{n-2} and F_{n-1} are terms in the sequence, correspondingly, they are also coefficients in F(x)
Noting this, I rewrite F(x) as
$F(x)=\cdots+F_{n-2}x^{n-2}+F_{n-1}x^{n-1}+F_nx^n+\cdots$
Like in the regular definition, I attempt to add F_{n-2} and F_{n-1}
The differing powers of x in F_{n-2} and F_{n-1} prevent me from grouping any like terms.
I should've expected it, of course. A generating function's terms are deliberately given different powers to prevent the coefficients from getting all mixed up.
Can't add them...


Although it's not as if I'm bad at math,
I realize I know so little.
When looking at all the math books here on these shelves,
Especially this part
And that's not just for math...

Likewise
The same applies to my understanding of Milka

Are you going to read all of them?
Thanks for your patronage!
She's very talented at mathematics.
Ignoring the material taught in our math classes, she busies herself in reading new material and problems that interest her.
Mmhmm.
I'm going to understand them
Huh?

I want to see what lies ahead
I wonder how vast the sight is before Milka...?

And not just in math either...
There's just no way I can keep up with Milka
Although I'm not too bad in mathematics myself,
"The key point is the eye."
The equations that effortlessly flow out from the tip of her fingers...
When I listen to her lectures
Her words reveal just how much my understanding is still lacking
"The mind's eye that can perceive the true form."
I can grasp the general concept but I can't fully understand it unless I work it out with my own two hands
But I wonder just how far I have to go until I can not only do that but also explain it to another person all the while in real-time...
I want to be able to reach that point where I, too, can chase after the proofs on my own without needing to consult references for every single step

If the powers of x are different, then I can always make them the same by multiplying x and using exponent laws.

$$x^{n-2} \cdot x^2 = x^{n-2+2} = x^n$$

For instance, $F_{n-2}x^{n-2}$ will be multiplied with x^2 to express it in terms of x^n. I apply the same process to $F_{n-1}x$ and $F_n x^n$ to get the following.

$$\begin{cases} F_{n-2}x^{n-2} \cdot x^2 = F_{n-2}x^n \\ F_{n-1}x^{n-1} \cdot x^1 = F_{n-1}x^n \\ F_{n-0}x^{n-1} \cdot x^0 = F_{n-0}x^n \end{cases}$$

But to get these terms, I would originally have to multiply the x to the entire function. Thus, I multiply x^0, x^1, and x^2 to F(x), giving me 3 new functions.

A $F(x) \cdot x^2 = F_0x^2 + F_1x^3 + F_2x^4 + \cdots$

B $F(x) \cdot x^1 = F_0x^1 + F_1x^2 + F_2x^3 + F_3x^4 + \cdots$

C $F(x) \cdot x^0 = F_0x^0 + F_1x^1 + F_2x^2 + F_3x^3 + F_4x^4 + \cdots$

With these new functions A, B, and C, I carry out A+B-C because it'll simplify things the most.

The above calculation will have the following left hand side.

$$\begin{aligned} \text{Left Hand Side} &= F(x) \cdot x^2 + F(x) \cdot x^1 - F(x) \cdot x^0 \\ &= F(x) \cdot (x^2 + x^1 - x^0) \end{aligned}$$

Meanwhile, the right hand side will look like this.

$$\begin{aligned} \text{Right Hand Side} &= F_0x^1 - F_0x^0 - F_1x^1 \\ &\quad + (F_0 + F_1 - F_2) \cdot x^2 \\ &\quad + (F_1 + F_2 - F_3) \cdot x^3 \\ &\quad + (F_2 + F_3 - F_4) \cdot x^4 \\ &\quad + \cdots \\ &\quad + (F_{n-2} + F_{n-1} - F_n) \cdot x^n \\ &\quad + \cdots \end{aligned}$$

To think the generating function for the Fibonacci sequence,

Would have such a simple and elegant form as this

In the right hand side, all the lines after the initial $F_0x^1-F_0x^0-F_1x^1$ take the form of $(F_{n-2}+F_{n-1}-F^n)x^n$. Since a number in the Fibonacci sequence is defined as Fn=Fn-1+Fn-2, all the terms inside the parentheses cancel out to 0

Thus, the equation simplifies to $F(x) \times (x^2+x^1-x^0) = F_0x^1-F_0x^0-F_1x^1$
By noting that $F_0=0$ and $F_1=1$, I can rewrite the entire equation to

$$F(x)\cdot(x^2+x-1) = -x$$

Dividing both sides by x^2+x-1, I thus obtain the following closed-form expression for $F(x)$.

$$F(x) = \frac{x}{1-x-x^2}$$

$$\langle 0, 1, 1, 2, 3, 5, 8, \cdots \rangle \leftrightarrow \frac{x}{1-x-x^2}$$

The beauty, power, and depth of mathematics...

"This one simple function houses the infinitely large Fibonacci sequence."
I've only just begun to get a taste for them.
"Compact, isn't it?"
Milka would surely say such a thing.
Closed-form expression of the Fibonacci Sequence's corresponding generating function
$F(x) = \frac{x}{1 - x - x^2}$
All I really want...

Is to see the world as she sees it.

So far, I've obtained the corresponding generating function for the Fibonacci sequence and obtained its closed-form expression. Now to express Fn in terms of n alone, I have to find the corresponding series to this closed-form generating function.

So my new goal now is...

$$\frac{x}{1-x-x^2}$$

Find a way to convert this expression back into an infinite series

"To find the general term,"

Cross back from the land of generating functions to the land of sequences to obtain the general term of the Fibonacci sequence.

I know that $\frac{1}{1-x}$ *has the power series expansion as shown below.*

$$\frac{1}{1-x} = 1+x+x^2+x^3+\cdots$$

The question is, can I convert $\frac{x}{1-x-x^2}$ *into a form similar to* $\frac{1}{1-x}$ *?*

"Its true form must first be perceived."

$$F(x) = \frac{x}{1-x-x^2}$$

The denominator $1-x-x^2$ is a quadratic equation, so I'll try to factorize it first.

Using r and s to represent some constant number, I can factorize $1-x-x^2$ into this.

$$1-x-x^2 = (1-rx)(1-sx)$$

This means that I can obtain the denominator $1-x-x^2$ by adding two fractions with the denominators $1-rx$ and $1-sx$ respectively.

$$\frac{1}{1-rx} + \frac{1}{1-sx} = \frac{(\text{Something})}{(1-rx)(1-sx)}$$
$$= \frac{(\text{Something})}{1-x-x^2}$$

Now, the problem is finding the exact values for r and s such that the numerator will become x

$$\frac{1}{1-rx} + \frac{1}{1-sx} = \frac{1-sx}{(1-rx)(1-sx)} + \frac{1-rx}{(1-rx)(1-sx)}$$
$$= \frac{2-(r+s)x}{1-(r+s)x+rsx^2}$$
$$= \cdots$$

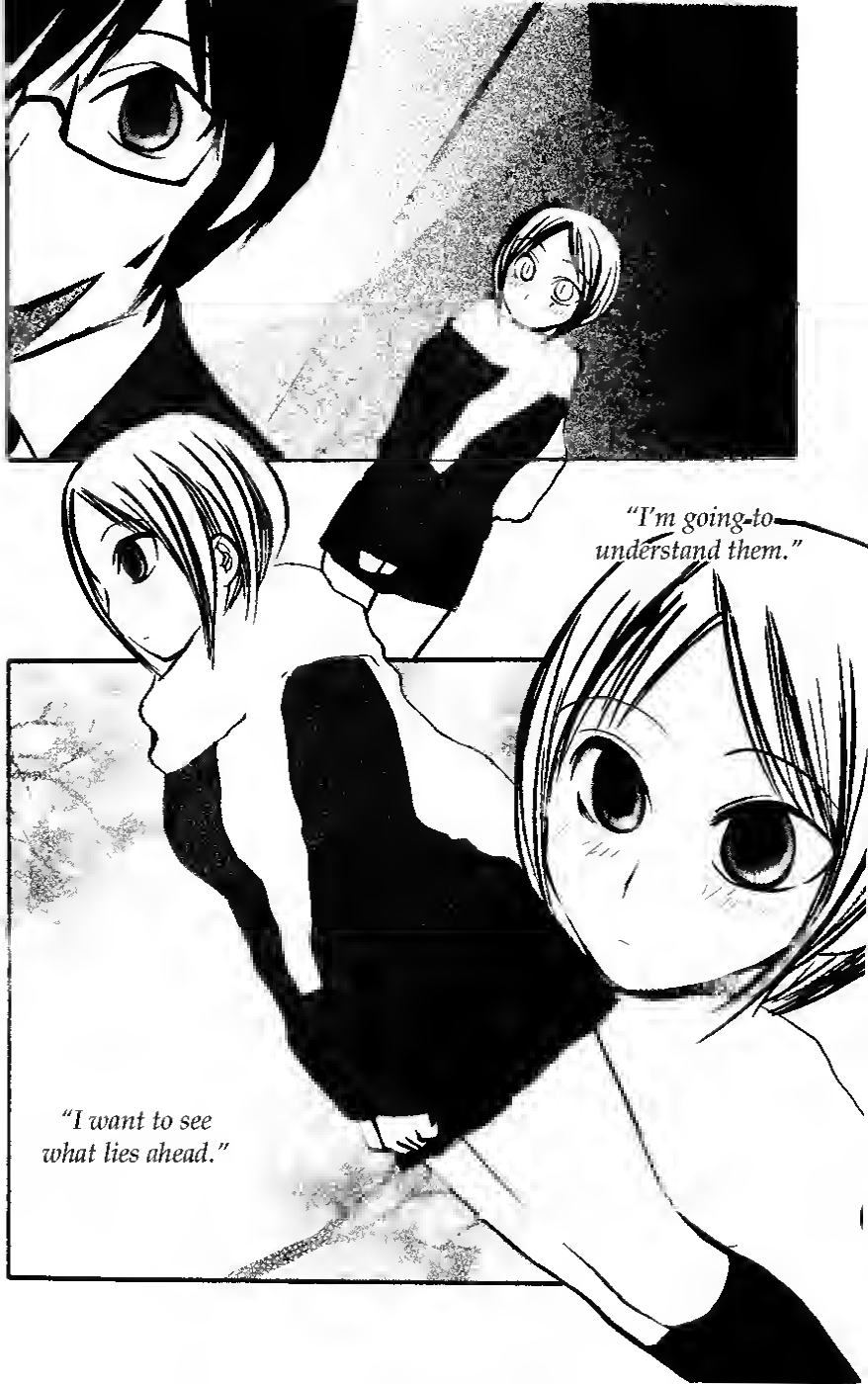
I know that with the right values of r and s, the denominator 1-(r+s)x+rsx² can be reduced to 1-x-x²
But there's no way that the numerator 2-(r+s)x can be reduced to x, since no matter the values of r and s, the 2 will be unaffected
I'm close...
But I'm stuck
LIBRARY

AH-
Yeah.
Piece being played by Milka...?
Is that...
Not → this

Just what exactly,
Is Milka...?
This is an animal
She's so amazing...
It's like she's good at practically everything,
Isn't she...
BOOK
Yeah, she is.

Compared to her,
"Perceive the true form,"
All I can manage is to tinker around with some formula
I feel as if I'm more than a hundred steps behind her...
"With the mind's eye."

I can't even catch the general term


"I'm going to understand them."
"I want to see what lies ahead."

T I P S

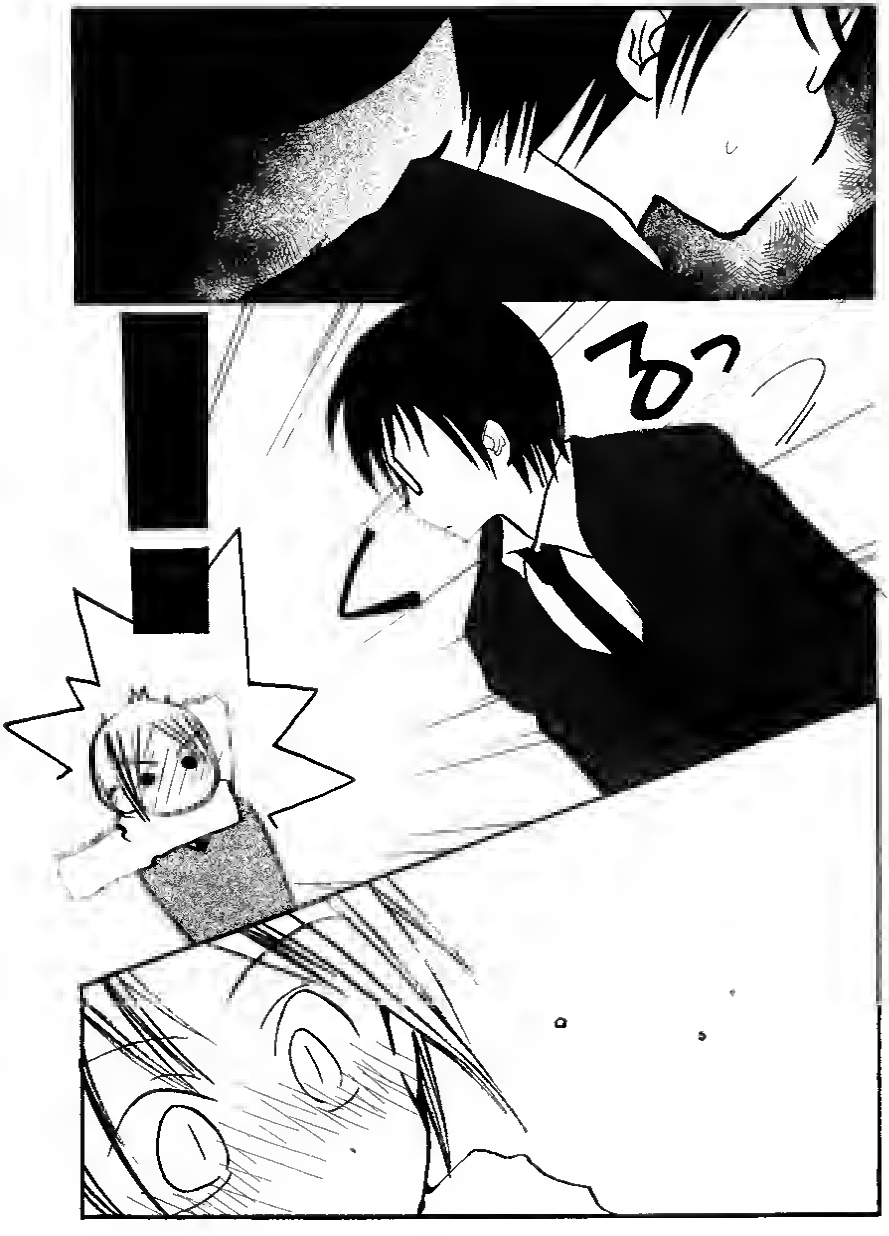
[*Picking a Book*]

Picking the right book is something that requires skill and thus should not be done carelessly. Flip through the book and make sure the material is interesting but also easy enough as to be on your level of comprehension. For beginners, start with short, thin books that are easy to finish before moving up to more challenging ones. Milko too, is quite selective in choosing right books to read, flipping with intense fury to see that it meets her standards.

数学ガール

Mathematical Girls

Chapter 13



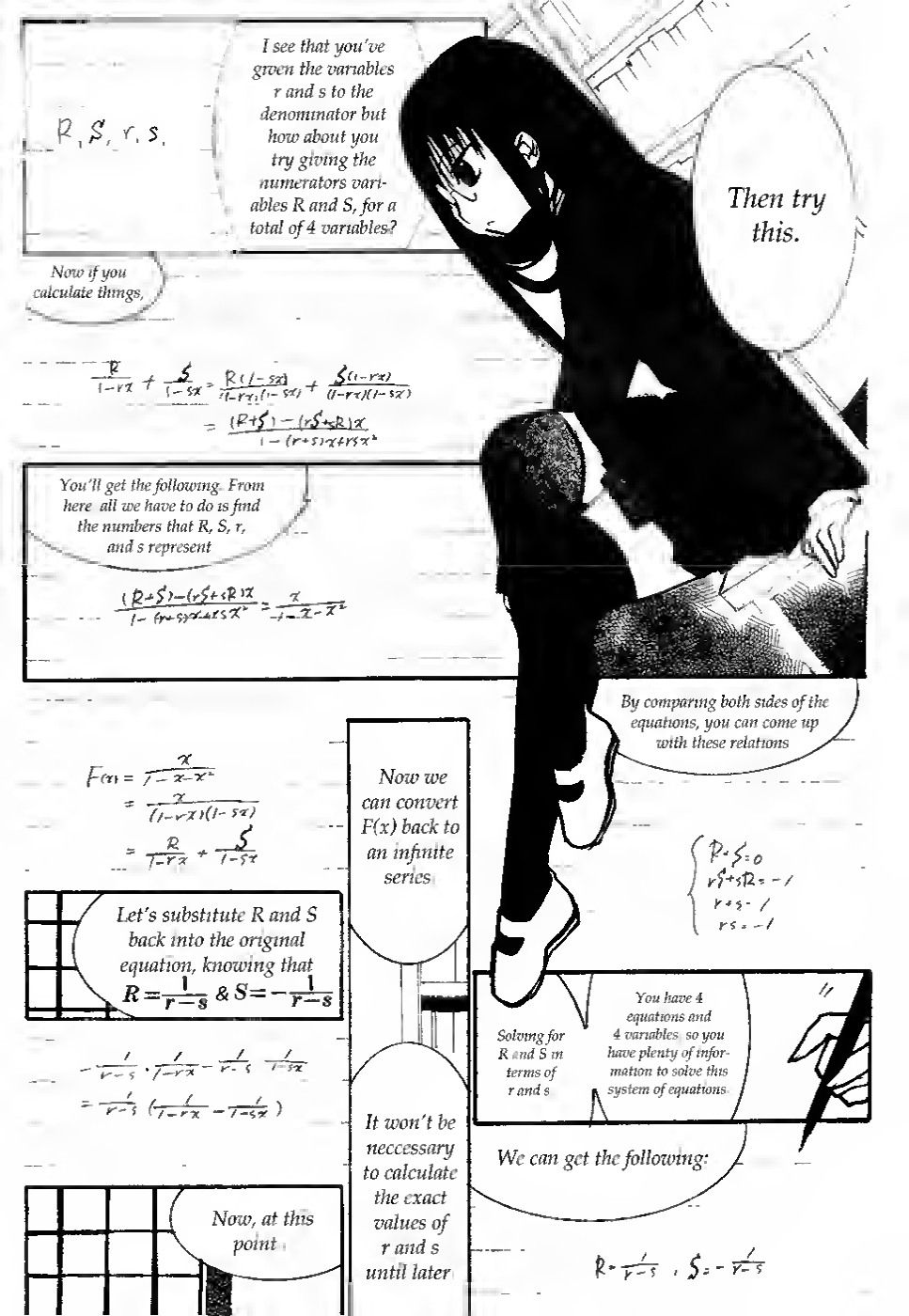
ぼう…

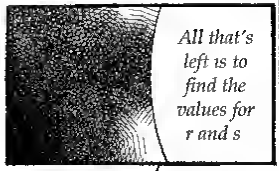
どた

ごご
ごろ

Are you alright?

Yes,
I'm fine...
LIBRARY
Well?
Oh?
I'm having a bit of trouble...


R, S, r, s.
I see that you've given the variables r and s to the denominator but how about you try giving the numerators variables R and S, for a total of 4 variables?
Then try this.
Now if you calculate things,
$\frac{R}{1-rx} + \frac{S}{1-sx} = \frac{R(1-sx)}{(1-rx)(1-sx)} + \frac{S(1-rx)}{(1-rx)(1-sx)}$
$= \frac{(R+S) - (rS+sR)x}{1-(r+s)x+rsx^2}$
You'll get the following. From here all we have to do is find the numbers that R, S, r, and s represent
$\frac{(R+S)-(rS+sR)x}{1-(r+s)x+rsx^2} = \frac{x}{1-x-x^2}$
By comparing both sides of the equations, you can come up with these relations
$\begin{cases} R+S=0 \\ rS+sR=-1 \\ r+s=1 \\ rs=-1 \end{cases}$
$F(x) = \frac{x}{1-x-x^2}$
$= \frac{x}{(1-rx)(1-sx)}$
$= \frac{R}{1-rx} + \frac{S}{1-sx}$
Now we can convert F(x) back to an infinite series
Let's substitute R and S back into the original equation, knowing that $R = \frac{1}{r-s}$ & $S = -\frac{1}{r-s}$
Solving for R and S in terms of r and s
You have 4 equations and 4 variables, so you have plenty of information to solve this system of equations.
$\frac{1}{r-s} \cdot \frac{1}{1-rx} - \frac{1}{r-s} \frac{1}{1-sx}$
$= \frac{1}{r-s}\left(\frac{1}{1-rx} - \frac{1}{1-sx}\right)$
It won't be neccessary to calculate the exact values of r and s until later
We can get the following:
$R = \frac{1}{r-s}$, $S = -\frac{1}{r-s}$
Now, at this point



This expression below We've finally obtained the general term for the Fibonacci sequence

$$F_n = \frac{r^n - s^n}{r-s}$$

We can expand by making use of

$$\frac{1}{1-rx} = 1 + rx + r^2x^2 + r^3x^3 + \cdots$$

$$\frac{1}{1-sx} = 1 + sx + s^2x^2 + s^3x^3 + \cdots$$

$$= \frac{1}{r-s}\left((1+rx+r^2x^2+r^3x^3+\cdots) - (1+sx+s^2x^2+s^3x^3+\cdots)\right)$$

$$= \frac{1}{r-s}\left((r-s)x + (r^2-s^2)x^2 + (r^3-s^3)x^3 + \cdots\right)$$

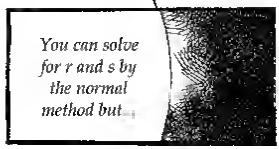
$$= \frac{r-s}{r-s}x + \frac{r^2-s^2}{r-s}x^2 + \frac{r^3-s^3}{r-s}x^3 + \cdots$$

Grouping it together like this, we can rewrite this as

$$F(x) = \underbrace{0}_{F_0} + \underbrace{\frac{r-s}{r-s}x}_{F_1} + \underbrace{\frac{r^2-s^2}{r-s}x^2}_{F_2} + \underbrace{\frac{r^3-s^3}{r-s}x^3}_{F_3} + \cdots$$

Previously, we knew that r and s are related by these two equations

$$\begin{cases} r+s=1 \\ rs=-1 \end{cases}$$



If you notice that their sum is 1 while their product is -1, you can quickly skip over to the quadratic equation $x^2-(r+s)x+rs=0$

$$x^2-(r+s)x+rs = (x-r)(x-s)$$

Since $r+s=1$ and $rs=-1$, the simplified quadratic equation is thus:

$$x^2-(r+s)x+rs = x^2-x-1 = 0$$

Using the quadratic formula would give you these roots.

$$x = \frac{1\pm\sqrt{5}}{2}$$

If we arbitrarily assume that $r>s$, then we assign the values as shown here

$$\begin{cases} r = \frac{1+\sqrt{5}}{2} \\ s = \frac{1-\sqrt{5}}{2} \end{cases}$$

Since $r-s=\sqrt{5}$

$$\frac{r^n-s^n}{r-s} = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$$

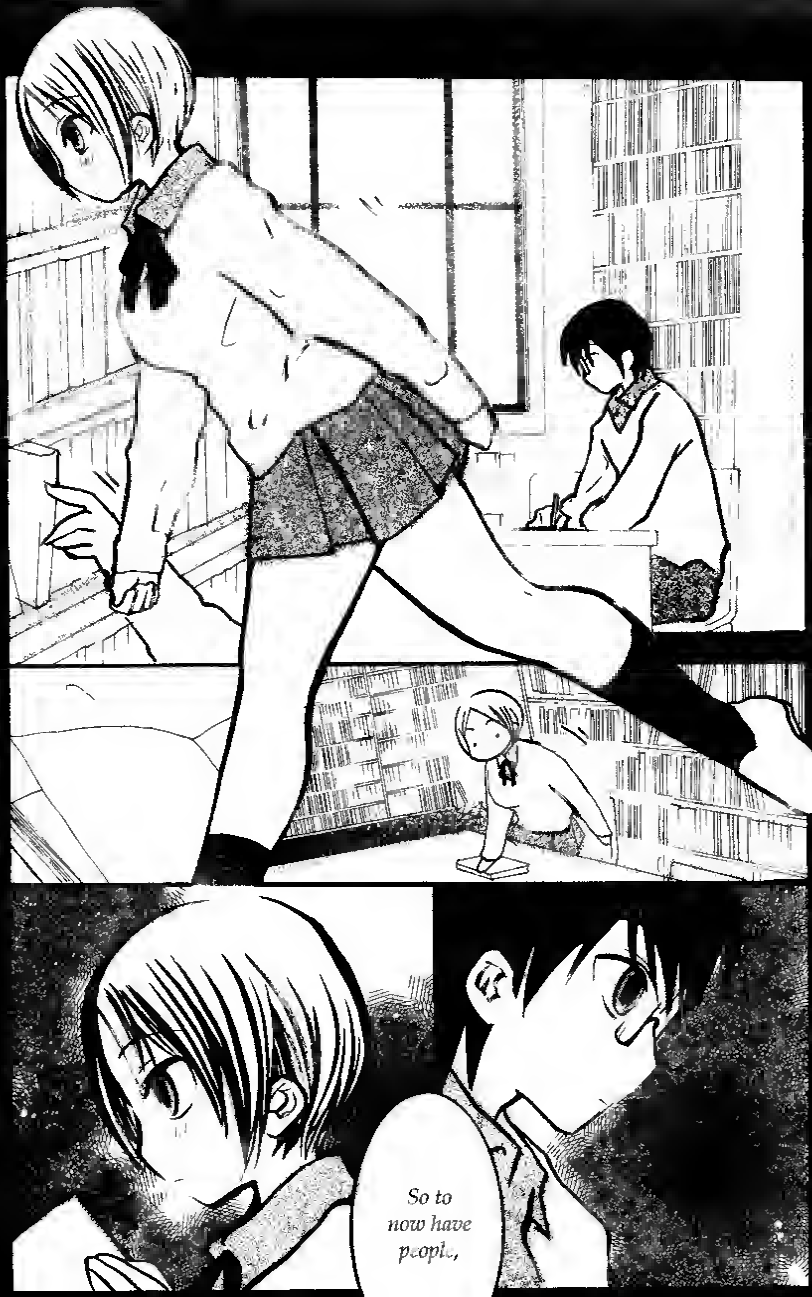
$F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$
We have the general term F_n of the Fibonacci sequence.
Back when you were a 3rd year in middle school and I was a 2nd year.
I remember you did a presentation for the culture festival.
About the binary number system.
And at the end of that presentation

For instance,
The 17th century mathematician Gottfried Leibniz would have had no idea what computers were
The work of early mathematicians who developed the system now lives on through modern-day computers.
You concluded that mathematics transcends time.
And though centuries have passed since his death
His mathematics has been passed on,
I too agreed with your conclusion from the bottom of my heart
To profoundly affect the modern man and his society.

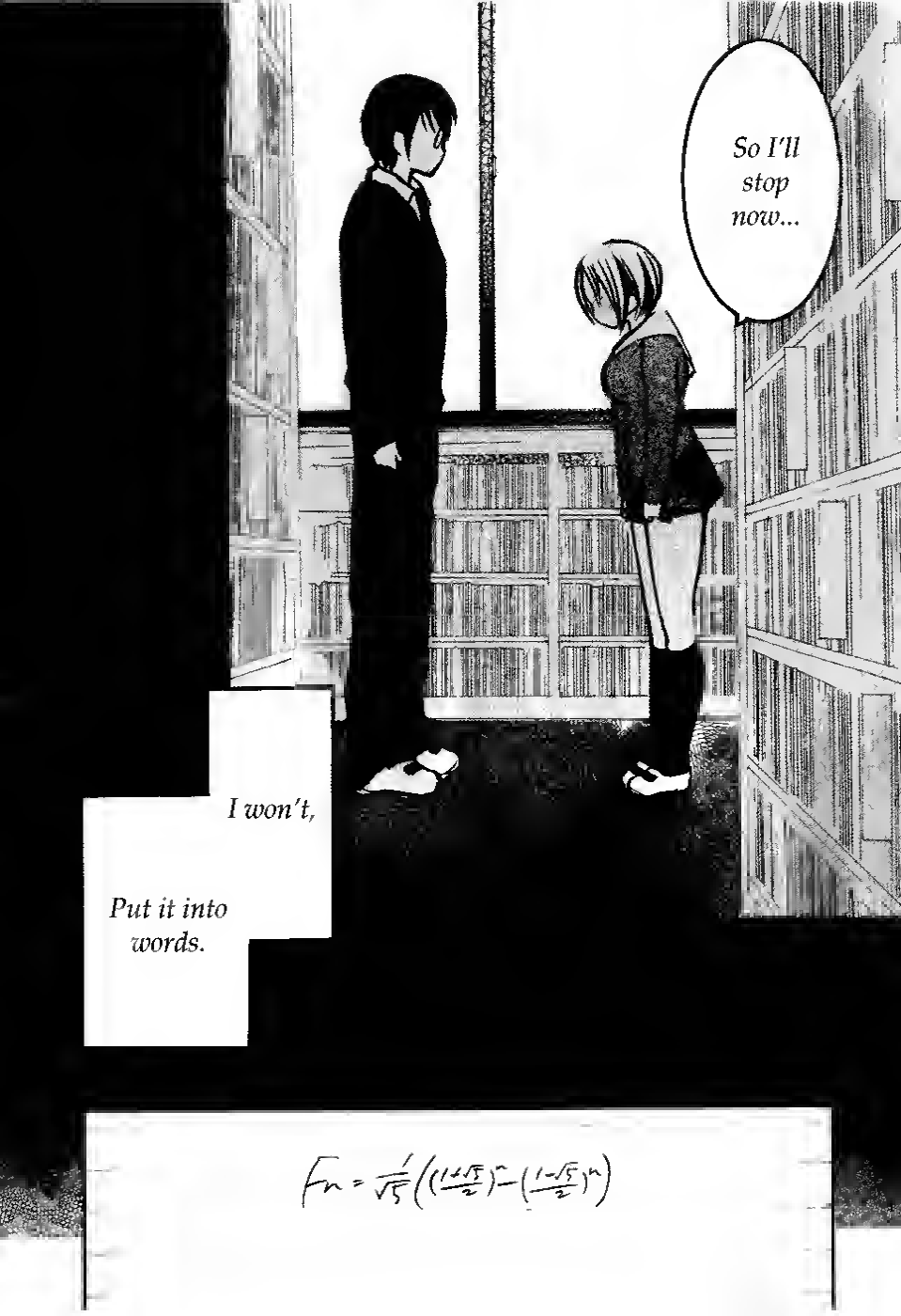
I'm glad I decided to write that letter to you back then.
Ever since then
I was even more glad when I was accepted to the same high-school you were at.
When I had entered highschool, it had become clear to me.
The phrase, "Mathematics transcends time,"
Has been etched across my heart.

But could not afford to.
The existence of words that I wanted to say.

I admire your honesty and how quickly you can understand things.
And,
Well, I personally find it very pleasant to talk with you, Tetra.
LIBRARY
Back in middle school
I was always studying by myself,
ひょ～


So to now have peopl-,

That I can talk freely about mathematics.
Is a lot more enjoyable.
Oh-
Thank you.
Umm
I...
Umm
I guess I've said too much for now.
I acciden tally went and even
Said some unnecessary things...


So I'll stop now...
I won't,
Put it into words.
$F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$

Really the general term for the Fibonacci sequence!?
I mean... With all these √5, does it even turn out an integer...?
This is...
Shall we check then?
What is it, Tetra?
Ah-
Umm...
I tried solving it too...
I wanted to see what the 100th term in the Fibonacci sequence would be like...

Half-way there!

One Line at a time

$F_{51} = 20365011074$

$F_{52} = 32951280099$

$F_{53} = 5331629117173$

$F_{54} = 86267571272$

$F_{55} = 139583862445$

$F_{56} = 22585143371$

$F_{57} = 36543529616 2$

$F_{58} = 59128677298779$

$F_{59} = 956722026041$

$F_{60} = 154800875592 0$

$F_{61} = 2504730781967$

$F_{62} = 4052739535 7881$

$F_{63} = 655747031984 2$

$F_{64} = 1061020985772 3$

$F_{65} = 1716768017756 5$

$F_{66} = 2777789003528 8$

$F_{67} = 4494557021285 3$

Calculating the Fibonacci Sequence:

START

$F_0=0$ $F_1=1$ $F_2=1$ $F_3=2$ $F_4=3$

$F_5=5$ $F_6=8$ $F_7=13$ $F_8=21$ $F_9=34$

$F_{10}=55$ $F_{11}=89$ $F_{12}=144$ $F_{13}=233$ $F_{14}=377$

$F_{15}=610$ $F_{16}=987$ $F_{17}=1597$ $F_{18}=2584$ $F_{19}=4181$

$F_{20}=6765$ $F_{21}=10946$ $F_{22}=17711$ $F_{23}=28657$

$F_{24}=46368$ $F_{25}=75025$ $F_{26}=121393$

$F_{27}=196418$ $F_{28}=317811$ $F_{29}=514229$

$F_{30}=832040$ $F_{31}=1346269$ $F_{32}=2178309$

$F_{33}=3524578$

One-third done!

Getting hard with such big numbers...

$F_{34}=5702887$ $F_{35}=9227465$ $F_{36}=14930352$

$F_{37}=24157817$ $F_{38}=39088169$

$F_{39}=63245986$ $F_{40}=102334155$

$F_{41}=165580141$ $F_{42}=267914296$

$F_{43}=433494437$ $F_{44}=701408733$

$F_{45}=1134903170$ $F_{46}=1836311903$

$F_{47}=2971215073$ $F_{48}=4807526976$

$F_{49}=7778742049$ $F_{50}=12586269025$

F_{86} = 4 2 0 1 9 6 1 4 0 7 2 7 4 8 9 6 7 3

F_{87} = 6 7 9 8 9 1 6 3 7 6 3 8 6 1 2 2 5 8

F_{88} = 1 1 0 0 0 8 7 7 7 8 3 6 6 1 0 1 9 3 1

F_{89} = 1 7 7 9 9 7 9 4 7 6 0 0 4 7 1 4 1 8 9

Just 10 more!

F_{90} = 2 8 8 0 0 6 7 1 9 4 3 7 0 8 1 6 1 2 0

F_{91} = 4 6 6 0 0 4 6 6 1 0 3 7 5 5 3 0 3 0 9

F_{92} = 7 5 4 0 1 1 3 8 0 4 7 4 6 3 4 6 4 2 9

F_{93} = 1 2 2 0 0 1 6 0 9 1 5 1 2 1 8 7 6 7 3 8

F_{94} = 1 9 7 4 0 2 7 4 2 1 9 8 6 8 2 2 3 1 6 7

F_{95} = 3 1 9 4 0 4 3 4 6 3 4 9 9 0 0 9 9 9 0 5

F_{96} = 5 1 6 8 0 7 0 8 8 5 4 8 5 8 3 2 3 0 7 2

F_{97} = 8 3 6 2 1 1 4 3 4 8 9 8 4 8 4 2 2 9 7 7

F_{98} = 1 3 5 3 0 1 8 5 2 3 4 4 7 0 6 7 4 6 0 4 9

F_{99} = 2 1 8 9 2 2 9 9 5 8 3 4 5 5 5 1 6 9 0 2 6

F_{100} = 3 5 4 2 2 4 8 4 8 1 7 9 2 6 1 9 1 5 0 7 5

GOAL!

Good job!

huff huff

F_{68} = 7 2 7 2 3 4 6 0 2 4 8 1 4 7

F_{69} = 1 1 7 6 6 9 0 3 0 4 6 0 9 9 4

F_{70} = 1 9 0 3 9 2 4 9 0 7 0 9 1 3 5

F_{71} = 3 0 8 0 6 1 5 2 7 1 7 0 1 2 9

F_{72} = 4 9 8 4 5 4 0 1 1 8 7 9 2 6 4

F_{73} = 8 0 6 5 1 5 5 3 3 0 4 9 3 9 3

F_{74} = 1 3 0 4 9 6 9 5 4 4 9 2 8 6 5 7

F_{75} = 2 1 1 1 4 8 5 0 7 7 9 7 8 0 5 0

Don't give up!

F_{76} = 3 4 1 6 4 5 4 6 2 2 9 0 6 7 0 7

F_{77} = 5 5 2 7 9 3 9 7 0 0 8 8 4 7 5 7

F_{78} = 8 9 4 4 3 9 4 3 2 3 7 9 1 4 6 4

F_{79} = 7 4 4 7 2 3 3 4 0 2 4 6 7 6 2 2 1

F_{80} = 2 3 4 1 6 7 2 8 3 4 8 4 6 7 6 8 5

F_{81} = 3 7 8 8 9 0 6 2 3 7 3 1 4 3 9 0 6

F_{82} = 6 1 3 0 5 7 9 0 7 2 1 6 1 1 5 9 1

F_{83} = 9 9 1 4 4 8 5 3 0 9 4 7 5 5 4 9 7

F_{84} = 1 6 0 5 0 0 6 4 3 8 1 6 3 6 7 0 8 8

Just a little bit more

F_{85} = 2 5 9 6 9 5 4 9 6 9 1 1 1 2 2 5 8 5

4 2 0 1 3 6 1 5 0 7 2 7 4 8 9 6 7 3

Mmhmm, it matches
That's great to hear!
I'm surprised you can just tell by looking...
But...
Hm?
Great work.
It's all thanks to your perseverance, Tetra.

Let's all give Tetra
A round of applause.
Uhh
Um
Be quiet in the library
.....

Thank you
so much...!

T I P S

[Gottfried Leibniz]

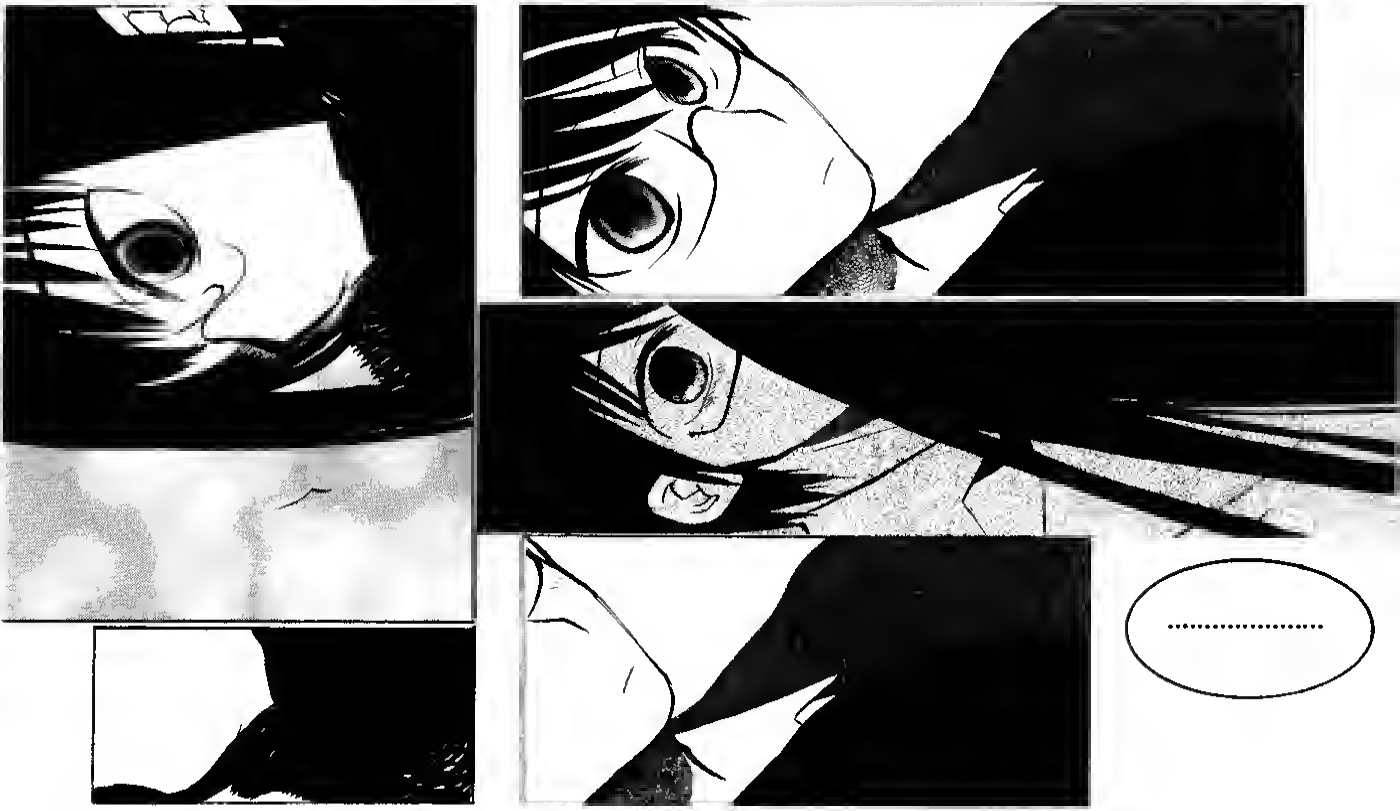
17th~18th century German mathematician and philosopher. Along with Newton, Leibniz is independently credited with the invention of calculus. Aside from calculus, Leibniz also advanced formal logic, mechanical calculators, and the idea of symbolic logic. His work on the binary number system has hugely contributed to the development of modern-day computers.

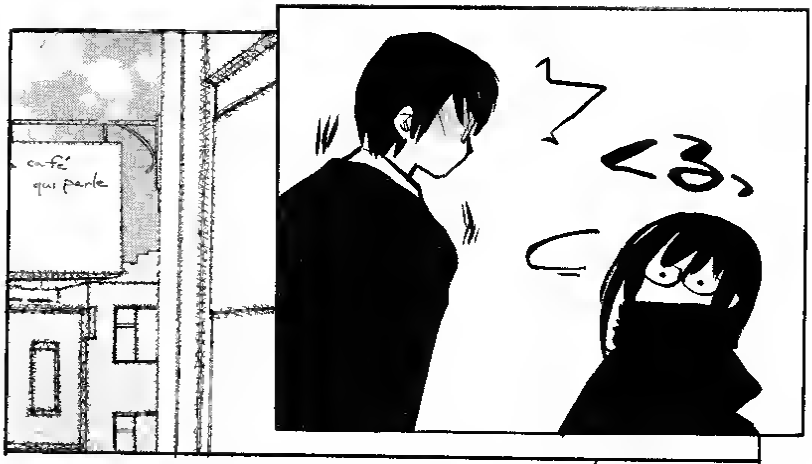
数学ガール

Mathematical Girls

Milka,
You're not wearing your muffler today
Aren't you cold?
Wha!?

..... Chapter 14 .....





General term for the Fibonacci sequence.

$$F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$$

Just in case, let's see what the value of F_n is when $n = 1,2,3,4$

$$F_0 = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^0 - \left(\frac{1-\sqrt{5}}{2}\right)^0\right) = \frac{0}{\sqrt{5}} = 0$$

$$F_1 = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^1 - \left(\frac{1-\sqrt{5}}{2}\right)^1\right) = \frac{\sqrt{5}}{\sqrt{5}} = 1$$

$$F_2 = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^2 - \left(\frac{1-\sqrt{5}}{2}\right)^2\right) = \frac{4\sqrt{5}}{4\sqrt{5}} = 1$$

$$F_3 = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^3 - \left(\frac{1-\sqrt{5}}{2}\right)^3\right) = \frac{11\sqrt{5}}{9\sqrt{5}} = 2$$

$$F_4 = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^4 - \left(\frac{1-\sqrt{5}}{2}\right)^4\right) = \frac{29\sqrt{5}}{16\sqrt{5}} = 3$$

I was tasked with finding the closed form expression of the general term for the Fibonacci sequence

That's amazing.

So as you can see, the √5s all cancel out nicely and we're left with our integer value

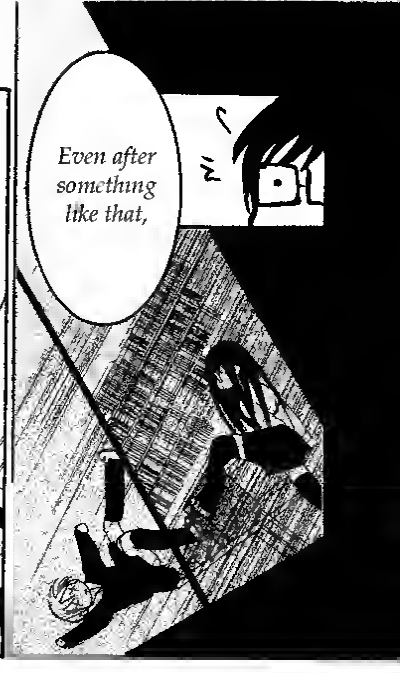
(1) *Find the corresponding generating function $F(x)$ for the sequence*
(2) *Find the closed-form expression of $F(x)$*
I used the recurrence relation of the Fibonacci sequence for this step
(3) *Convert the closed-form expression of $F(x)$ into an infinite series*
After doing so, the coefficients of x^n was the general term of the Fibonacci sequence

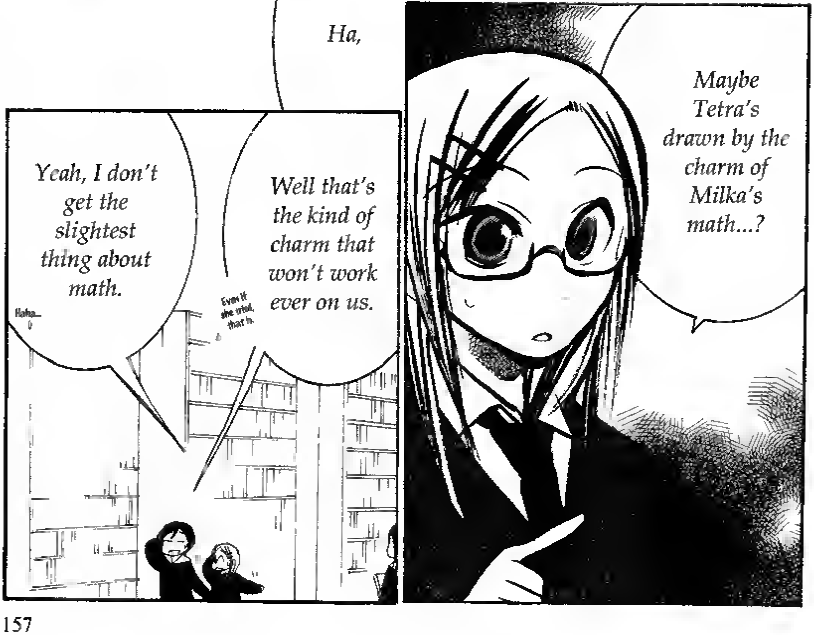
And in doing so, I went through these main steps

"Itinerary from a sequence to its general term expression"
Sequence
(1)
Generating Function
(2)
General term of Sequence
Closed-Form Expression of Generating Function
Even so, generating functions are a powerful tool for investigating sequences
Still seems like a rather long roundabout way
So we were able to catch the sequence by utilizing its generating function, whose coefficients form the original sequence
Yes, it is bad if you don't state specific conditions, that is
But isn't it bad to change the order of terms in an infinite series since doing so could potentially lead to a different sum?
That's because we already have so many ways of manipulating functions By crossing over from the land of sequences to the land of functions, all of our function-manipulating methods can now be of use
But this time, it's fine
We'll keep it a secret between us that we used a generating function and instead inductively prove that our expression for the general term does indeed give the original sequence
.....

I still can't understand...
Close, aren't they...

Hmm...
It'd normally be impossible even if they tried.
How could those two get along at all?


Even after something like that,

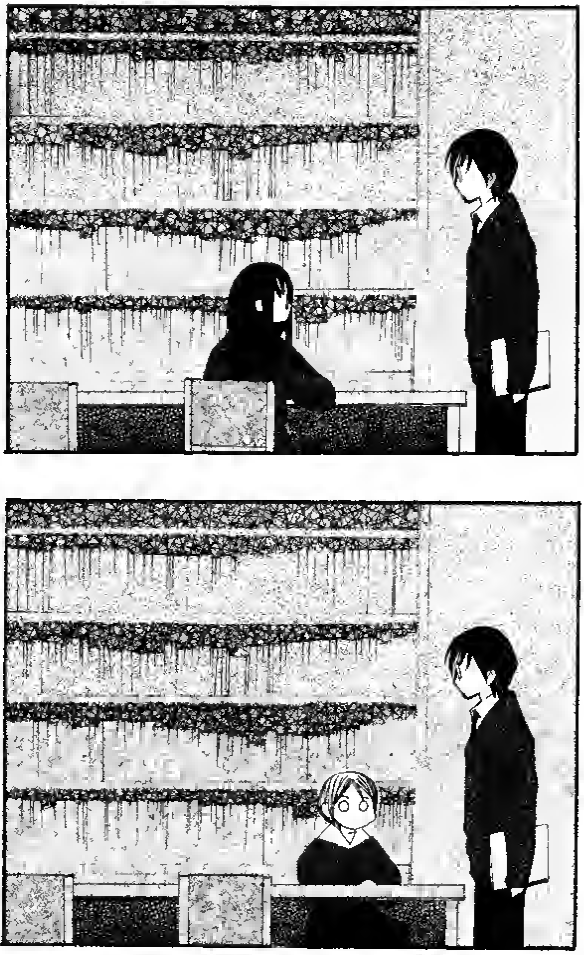

Ha,
Maybe Tetra's drawn by the charm of Milka's math...?
Yeah, I don't get the slightest thing about math.
Well that's the kind of charm that won't work ever on us.
Even if she tried, that is.
Haha...

ばったり。
Math and I
I'd never even think about trying to understand it!
DO NOT get along!

I WON'T EVER... YOU HEAR!?
Quiet in the library!
"Could you just..."
Could be seen as a rejection
But even so. .
I suppose that time back then,

I suppose nothing's really changed in the end
"Continue to teach me math as usual?"
Because of that,

If...



There were only two people in this world,

Would our worries really be less...?

.....
Uhh yeah, just wondering what it'd be like if there were only two people in this world...
Thinking about something?
Hmm...
Why don't we change the topic to what would happen if there were only two prime numbers in this world?

First, consider the product

$(2^0+2^1+2^2+\cdots)(3^0+3^1+3^2+\cdots)$

This product would diverge to positive infinity but let's try expanding the first few terms

$2^0 3^0 + 2^0 3^1 + 2^1 3^0 + 2^0 3^2 + 2^1 3^1 + 2^2 3^0 + \cdots$

If we group the terms purely based on the sum of their exponents, we see a pattern emerge

$(2^0 3^0) + (2^0 3^1 + 2^1 3^0) + (2^0 3^2 + 2^1 3^1 + 2^2 3^0) + \cdots$

In short, we can summarize this as

$$\sum_{n=0}^{\infty} \sum_{k=0}^{n} 2^k 3^{n-k}$$

Let's take it one step at a time.

Oh, this is a convolution. In the outer sigma $\sum_{n=0}^{\infty}$, n increases from 0, 1, 2, ... , to add the terms in the inner sigma, $\sum_{k=0}^{n}$, which in turn, is adding the terms whose exponents add up to n. Alright, I get it.

So basically what we're doing is partitioning the product into exponents of 2 and 3

Partition?

Well, I suppose that's one way to put it.

Now in this $2^k 3^{n-k}$, every positive integer which has the prime factor 2 or 3 should appear at least once since the exponents can be any integer greater than or equal to 0.

Right.

Of course, the prime number 1 will show up as well even though it doesn't have 2 nor 3 as a factor.

Now let's consider the product of the infinite series below. We'll call this Q_2.

$$Q_2 = \left(\frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \cdots\right)\left(\frac{1}{3^0} + \frac{1}{3^1} + \frac{1}{3^2} + \cdots\right)$$

In the previous case, our product diverged to positive infinity. However, this time it's different because the two infinite series that are multiplied together are convergent geometric series.

$$Q_2 = \left(\frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \cdots\right)\left(\frac{1}{3^0} + \frac{1}{3^1} + \frac{1}{3^2} + \cdots\right)$$

$$= \left(\frac{1}{1-\frac{1}{2}}\right) \cdot \left(\frac{1}{1-\frac{1}{3}}\right) \quad \text{< Product Form >}$$

Let's try expanding the first few terms of Q_2. After rewriting the expanded form into a summation form with sigma, we see that the $2^k 3^{n-k}$ we saw earlier is a denominator this time.

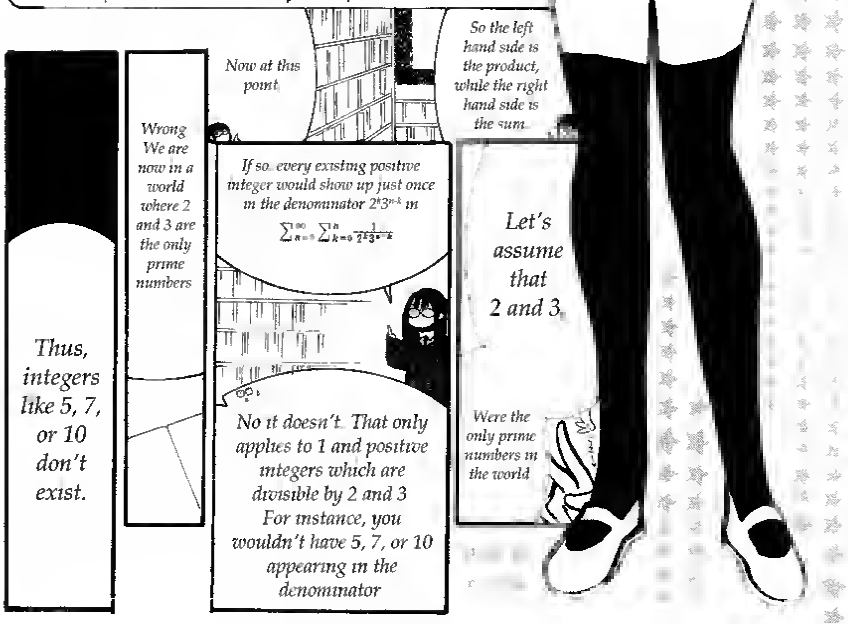
$$Q_2 = \left(\frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \cdots\right)\left(\frac{1}{3^0} + \frac{1}{3^1} + \frac{1}{3^2} + \cdots\right)$$

$$= \underbrace{\left(\frac{1}{2^0 3^0}\right)}_{n=0} + \underbrace{\left(\frac{1}{2^0 3^1} + \frac{1}{2^1 3^0}\right)}_{n=1} + \underbrace{\left(\frac{1}{2^0 3^2} + \frac{1}{2^1 3^1} + \frac{1}{2^2 3^0}\right)}_{n=2} + \cdots$$

$$= \sum_{n=0}^{\infty} \sum_{k=0}^{n} \frac{1}{2^k 3^{n-k}} \quad \text{< Summation Form >}$$

With these two forms for Q_2, we can now state the following!

$$\left(\frac{1}{1-\frac{1}{2}}\right)\left(\frac{1}{1-\frac{1}{3}}\right) = \sum_{n=0}^{\infty} \sum_{k=0}^{n} \frac{1}{2^k 3^{n-k}}$$


So the left hand side is the product, while the right hand side is the sum.
Let's assume that 2 and 3 are the only prime numbers in the world
Were the only prime numbers in the world
Now at this point
If so, every existing positive integer would show up just once in the denominator $2^k 3^{n-k}$ in $\sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{1}{2^k 3^{n-k}}$
No it doesn't. That only applies to 1 and positive integers which are divisible by 2 and 3. For instance, you wouldn't have 5, 7, or 10 appearing in the denominator
Wrong. We are now in a world where 2 and 3 are the only prime numbers
Thus, integers like 5, 7, or 10 don't exist.

Proof by Contradiction

Also known as reductio ad impossibilem, this is one way of proving a proposition. First, one assumes that the negation of the proposition in question is true. Then, by showing that this leads to a contradiction, one indirectly proves the original proposition. Since this is a rather tricky way that requires contradicting the opposite, many people have trouble with it.

Proof by Contradiction

Assumption: A statement's opposite is true

↓

Find Contradiction

↓

The Opposite cannot be true

↓

Result: The statement is true

Oh, so we're proving by contradiction?

That's according to the unique prime factorization theorem.

Since every integer greater than 1 can be written as a unique product of prime numbers,

If 2 and 3 were the only prime numbers, then it follows that integers like 5 or 7 cannot exist.

Now although I understand that

I can't quite fully accept it.

Alright then.

If it's the fact there's only 2,

Let's make it so that there's m-number of primes instead.

No,

Exactly. I'm assuming that the number of primes is finite.

Whether or not there are 2 primes or m primes assuming so would make the number of primes finite.

We'll first assume that the number of primes is finite so we can prove by contradiction that there is actually an infinite number of primes.

Assume that there are m primes. By writing them out, we have

$$p_1, p_2, \cdots, p_k = p_m$$

The first three terms in this list are $p_1=2$, $p_2=3$, $p_3=5$. Now let us consider the finite product Q_m, which results from multiplying the sums of infinite series as shown below.

$$Q_m = \left(\frac{1}{2^0}+\frac{1}{2^1}+\frac{1}{2^2}+\cdots\right)\left(\frac{1}{3^0}+\frac{1}{3^1}+\frac{1}{3^2}+\cdots\right)\left(\frac{1}{p_m^0}+\frac{1}{p_m^1}+\frac{1}{p_m^2}+\cdots\right)$$

$$= \prod_{k=1}^{m}\left(\frac{1}{p_k^0}+\frac{1}{p_k^1}+\frac{1}{p_k^2}+\cdots\right)$$

$$= \prod_{k=1}^{m}\frac{1}{1-\frac{1}{p_k}} \quad \text{< Product Form >}$$

All we're doing is making our Q_2 from before, which assumed that there were only 2 primes, into Q_m, which now assumes that there are m primes. Since we're multiplying a finite number of factors, which is m, it must follow that the product Q_m is finite as well.

That by adding 1 to the product of a finite number of primes-

So are we then using Euclid's method?

We first assume that the number of primes is finite

No

Now here's the interesting part. Let's try expanding this product just like how we did previously when 2 and 3 were the only prime numbers

Yes, good

UHHH ?

Right, right. Since p_k is at least as great as 2, the infinite geometric series $\frac{1}{p_k^0}+\frac{1}{p_k^1}+\frac{1}{p_k^2}+\cdots$ would converge to the finite number $\frac{1}{1-\frac{1}{p_k}}$

There's nothing written on it but understand the second sigma as signifying the addition of terms in which the exponents $r_1, r_2, \cdots, r_m$ add up to n

Not sure if I understand this I mean, the second sigma doesn't even have anything written above or below it

To use your words, that is.

This time, we'll 'partition' this into products of not two, but m factors,

Q_m can be thus seen as the sum of terms which all take the form of 1/(product of primes), and this product is as written below.

$$2^{r_1} 3^{r_2} 5^{r_3} \cdots p_m^{r_m}$$

Oh, so we're basically grouping together the terms whose exponents add up to n like before

Exactly. Now then,

$$Q_m = \left(\frac{1}{2^0}+\frac{1}{2^1}+\frac{1}{2^2}+\cdots\right)\left(\frac{1}{3^0}+\frac{1}{3^1}+\frac{1}{3^2}+\cdots\right)\left(\frac{1}{p_m^0}+\frac{1}{p_m^1}+\frac{1}{p_m^2}+\cdots\right)$$

$$= \underbrace{\left(\frac{1}{2^0 3^0 5^0 \cdots p_m^0}\right)}_{\text{Sum of the exponents is 0}} + \underbrace{\left(\frac{1}{2^1 3^0 5^0 \cdots p_m^0} + \cdots + \frac{1}{2^0 3^0 5^0 \cdots p_m^1}\right)}_{\text{Sum of the exponents is 1}} + \cdots$$

$$= \sum_{n=0}^{\infty}\underbrace{\sum \frac{1}{2^{r_1} 3^{r_2} 5^{r_3} \cdots p_m^{r_m}}}_{\text{Sum of the exponents is } n} \quad \text{< Product Form >}$$

And so we get this.

Previously, we assumed that there were only m number of primes. Thus, according to the unique prime factorization theorem, every positive integer that exists can be written as a unique product of primes, which takes the form of $p_1^{r_1} p_2^{r_2} p_3^{r_3} \ldots p_m^{r_m}$. In other words, since the terms of Q_m all take the form of 1/(product of primes), every positive integer will appear just once in the denominator.

Again, this is exactly like in the previous case with 2 and 3 as our only prime numbers.

Now, if every positive integer appears once in the denominator, then we can rewrite Q_m as shown below

$$Q_m = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$$

On the left hand side, we have a finite number, while on the right hand side, we have a series that diverges to infinity. We've now found our contradiction.

!

Our initial assumption, that there are a finite number of prime numbers, has now been disproven.

Hence, the opposite, that there are infinitely many prime numbers, is proven true.

Previously, we showed that Qm must be finite by observing that it was a simply a product of convergent geometric series, which are finite numbers themselves.

$$Q_m = \prod_{k=1}^{m} \frac{1}{1-\frac{1}{k}} \quad \text{(Finite Number)}$$

However, we are now able to show that Qm is also equal to the harmonic series $\sum_{k=1}^{\infty} \frac{1}{k}$

$$Q_m = \sum_{k=1}^{\infty} \frac{1}{k} \quad \text{(Harmonic Series)}$$

In short, what we have is this.

$$\prod_{k=1}^{m} \frac{1}{1-\frac{1}{k}} = \sum_{k=1}^{\infty} \frac{1}{k}$$

So you noticed?

That's a harmonic series!

As you know, the harmonic series diverges to positive infinity.

Using the harmonic series to prove the infinity of prime numbers...

This brilliant proof is a page taken from our master's book.

Quod Erat Demonstrandum
- Q.E.D. -

Harmonic Series and the Infinitude of Primes

Assume finitude of primes

↓

< Product Form > ← ∞ → < Summation Form >

↓ ↓

$\prod_{k=1}^{n} \frac{1}{1-\frac{1}{p_k}}$ $\sum_{k=1}^{\infty} \frac{1}{k}$

↓ ↓

Converges → Contradiction ← Diverges

↓

Assumption is false

↓

There are infinitely many primes

Our master...?

Our master,
Was once described with the quote,
"He calculated without any apparent effort, just as men breathe, as eagles sustain themselves in the air."

And that man is 18th century's greatest mathemati-cian,
Leonhard Euler.
...For a short while
I almost thought I alone got the wrong idea about that earlier time

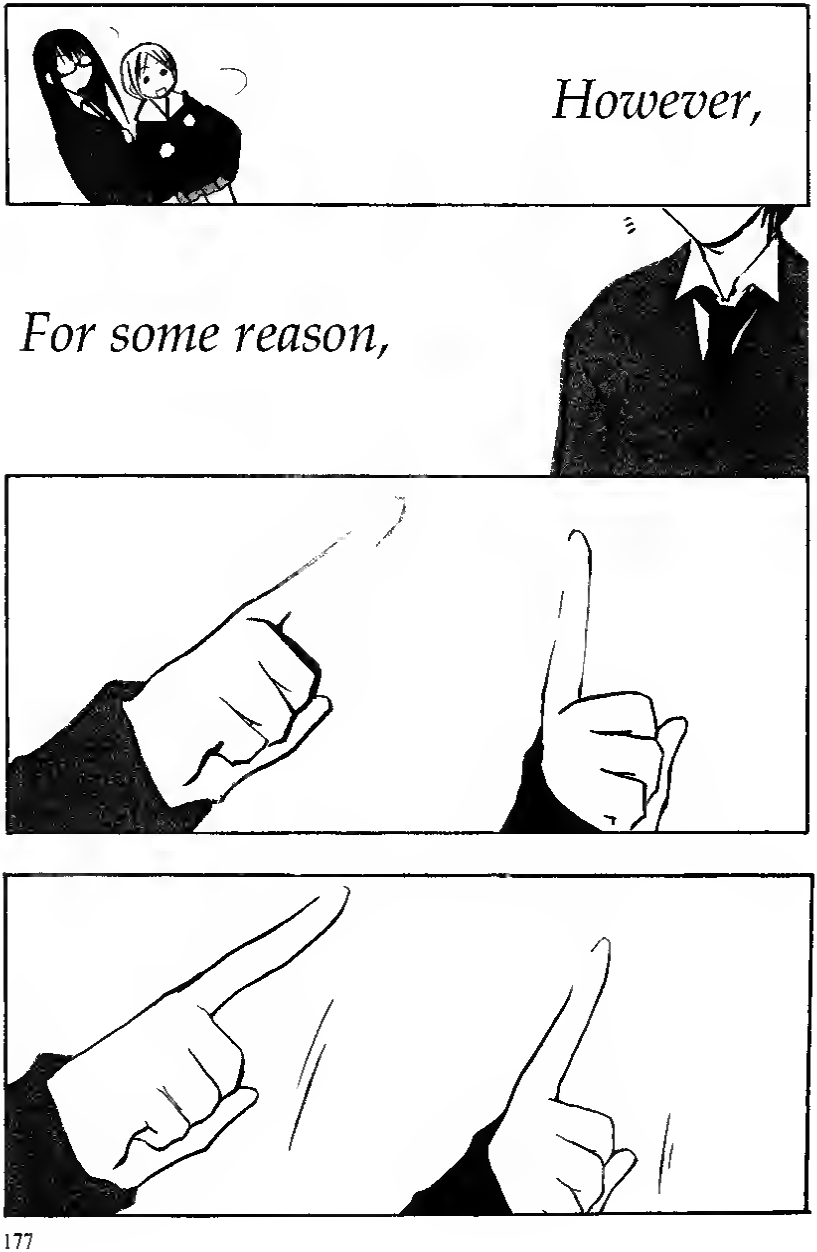
Well, we could remain partitioned,
But I'd rather not.
Sorry to keep you waiting!
Let's go home!
Twitch
Twitch
Nadeshiko

A little earlier,
Tetra,
How exactly did you come to get along so well with Milka?
I tried directly asking Tetra.
I mean,
You know...
There is some history between you two...
Not that I think your new friendship
Is a bad thing or anything but I'm just curious

Oh...
I'm sure,
Asking Milka too would give me a similar answer...
.....
It's hard to know just what exactly they're thinking,
Much less understand them...
Huh?
That's a
SECRET!

Right!?
We've decided to drop by the bookstore today
Oh
Tetra, isn't your home... the other way?
Then I'll see you tomorrow
Let's do more math together tomorrow
Alright, Senpai?
Like attempting to solve a difficult math problem...

...It's almost quite frustrating.
The answer,
Won't come easily.
Even getting a clue is difficult.


However,
For some reason,

That aspect,

Makes it all the more charming,

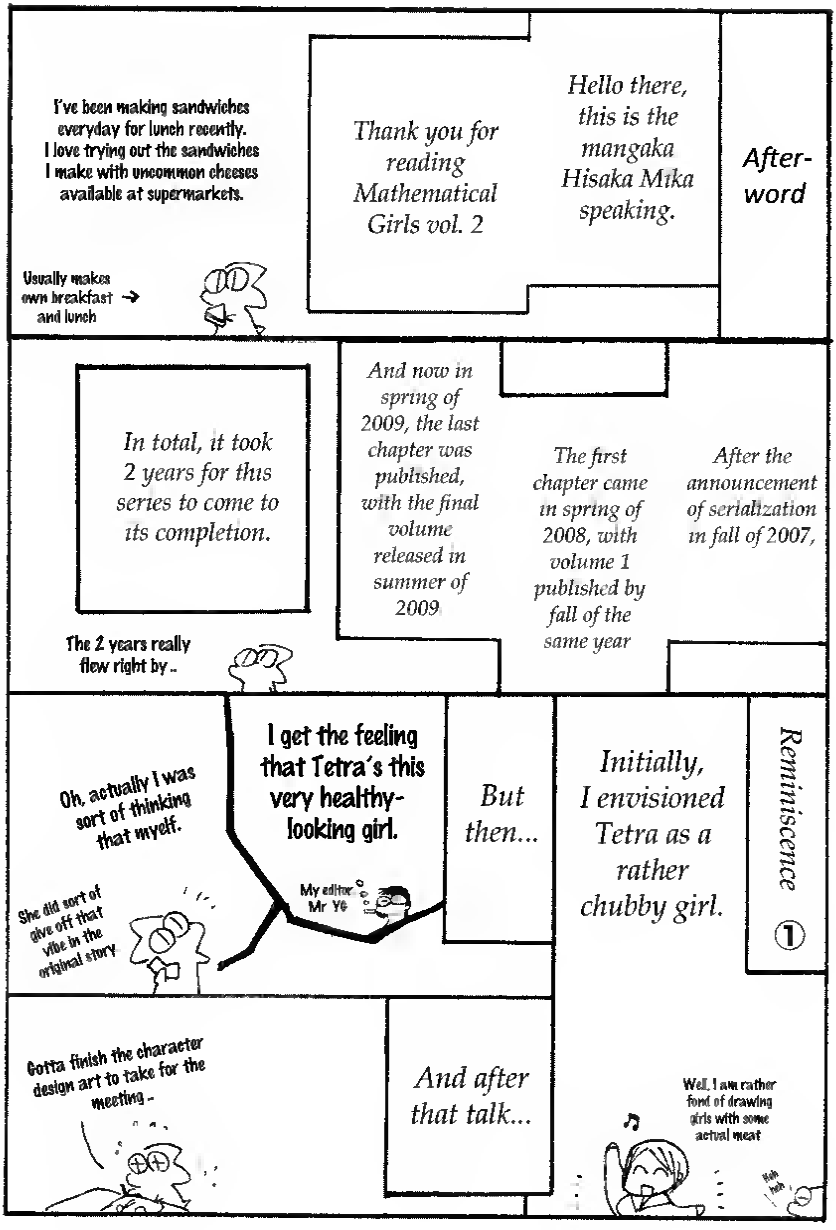
And
unforgettable...

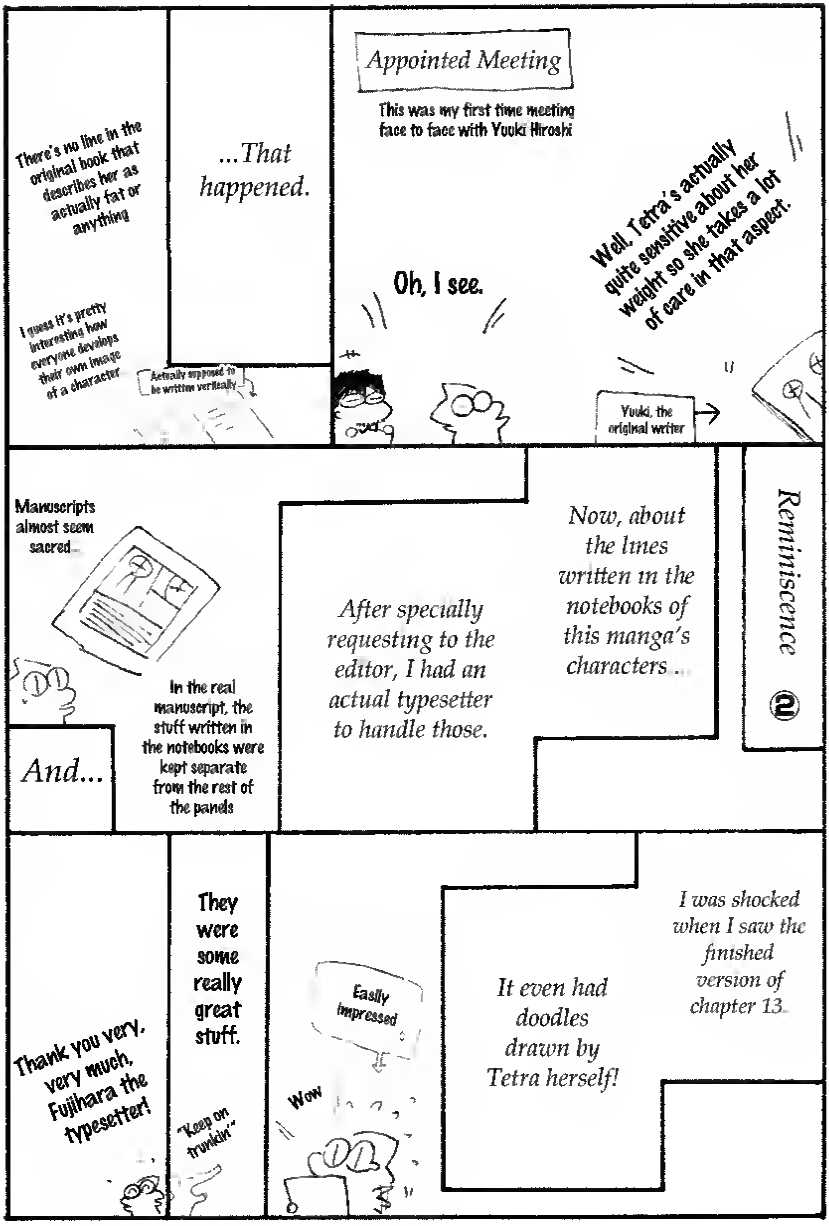
Mathematical Girls
-End-

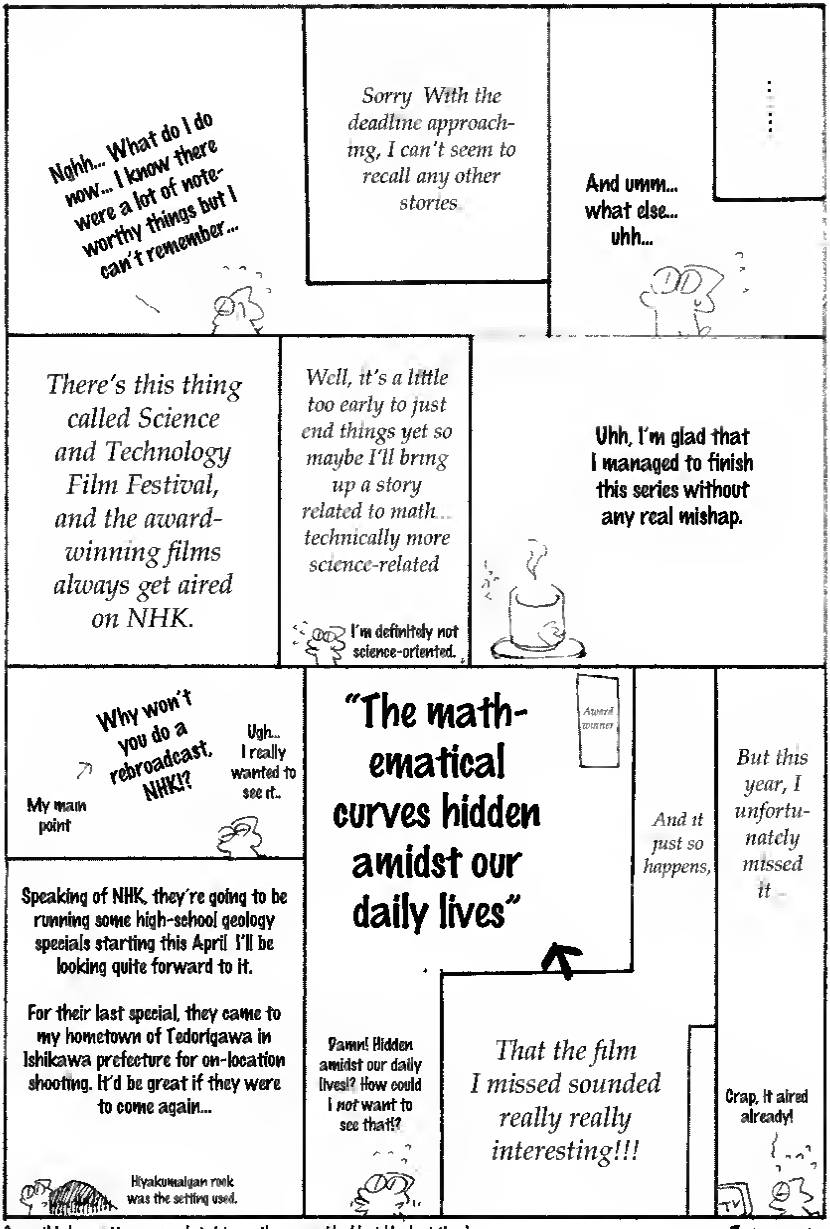
T I P S

[Leonhard Euler]

One of the greatest mathematicians not just in the 18th century, but in the whole of human history. He made far-reaching contributions to mathematical analysis, number theory, geometry, and physics. Although best remembered for his famous "Euler's identity," he formulated countless of other formulae and theorems. He suffered from near total blindness during the latter years of his life but this did nothing to hamper his productivity. Milka refers to Euler as "master."


I've been making sandwiches everyday for lunch recently. I love trying out the sandwiches I make with uncommon cheeses available at supermarkets.
Usually makes own breakfast and lunch
Thank you for reading Mathematical Girls vol. 2
Hello there, this is the mangaka Hisaka Mika speaking.
After-word
In total, it took 2 years for this series to come to its completion.
And now in spring of 2009, the last chapter was published, with the final volume released in summer of 2009
The first chapter came in spring of 2008, with volume 1 published by fall of the same year
After the announcement of serialization in fall of 2007,
The 2 years really flew right by..
Oh, actually I was sort of thinking that myself.
I get the feeling that Tetra's this very healthy-looking girl.
My editor Mr Ye
She did sort of give off that vibe in the original story
But then...
Initially, I envisioned Tetra as a rather chubby girl.
Reminiscence ①
Gotta finish the character design art to take for the meeting -
And after that talk...
Well, I am rather fond of drawing girls with some actual meat
Nah nah


There's no line in the original book that describes her as actually fat or anything
I guess it's pretty interesting how everyone develops their own image of a character
...That happened.
Actually supposed to be written vertically
Appointed Meeting
This was my first time meeting face to face with Yuuki Hiroshi
Oh, I see.
Well, Tetra's actually quite sensitive about her weight so she takes a lot of care in that aspect.
Yuuki, the original writer
Reminiscence 2
Now, about the lines written in the notebooks of this manga's characters...
After specially requesting to the editor, I had an actual typesetter to handle those.
Manuscripts almost seem sacred...
In the real manuscript, the stuff written in the notebooks were kept separate from the rest of the panels
And...
I was shocked when I saw the finished version of chapter 13.
It even had doodles drawn by Tetra herself!
Easily impressed
Wow
They were some really great stuff.
"Keep on truckin'"
Thank you very, very much, Fujihara the typesetter!


Nghh... What do I do now... I know there were a lot of note-worthy things but I can't remember...
Sorry. With the deadline approaching, I can't seem to recall any other stories.
And umm... what else... uhh...
There's this thing called Science and Technology Film Festival, and the award-winning films always get aired on NHK.
Well, it's a little too early to just end things yet so maybe I'll bring up a story related to math... technically more science-related
I'm definitely not science-oriented.
Uhh, I'm glad that I managed to finish this series without any real mishap.
Why won't you do a rebroadcast, NHK!?
My main point
Ugh... I really wanted to see it..
"The mathematical curves hidden amidst our daily lives"
Award winner
And it just so happens,
But this year, I unfortunately missed it.
Speaking of NHK, they're going to be running some high-school geology specials starting this April. I'll be looking quite forward to it.
For their last special, they came to my hometown of Tedorigawa in Ishikawa prefecture for on-location shooting. It'd be great if they were to come again...
Hiyakumaigan rock was the setting used.
Damn! Hidden amidst our daily lives!? How could I not want to see that!?
That the film I missed sounded really really interesting!!!
Crap, it aired already!
TV
Aired earlier that very day

Guess this has gotten as unrelated to math as possible. (Just like last time)

★And so, a thank you to all the staff. ★

There are still lots of people I have to thank outside the ones listed here.

I really owe a lot to you folks for making this manga possible.

Yuuki Hiroshi (Original Writer, also checked the math)

Yurugi Sou (Editor / Involved in 14 other series)

Fujihara Mimi (Typesetter)

Nawada Kouhei Design Co **Nawada Kouhei** (cover design)

Kaneko Yasoko (art assistant)

Tom-neko . (art assistant)

Data Collection / The folks at Ishikawa Pref. Children's Activity Center

Mouri

And last but not least, you, the readers, for reading this manga.

Thank you all very much!!!

I still have lots of room for improvement so I'll keep trying my best!

This is Hisaka Mika signing off!

2009.6

This manga was really quite the learning experience for me. I doubt I could have done it alone.

May we meet again in the near future.

Well then, that's it for now.

Afterword

by Yuuki Hiroshi

Hello there. This is the writer Yuuki Hiroshi speaking. Thank you far reoding Mathematicol Girls vol. 2. Bock when this mango first storted its seriolizotion in Comic Flapper Magazine, I remember myself checking if the math were correct oll the while being overwhelmed with bittersweet feelings os I thought, "Milko's so stunning... Tetro's so cute... I'm so jealous of 'me,' the protogonist..."

In the original stary, I wrote it solely from the perspective of "me," so it's rother hard to understond what's going through other choracters' minds. However, in this manga-adaptatian, there ore moments narroted by Tetro and I thought thot helped a lot in creating a wider ond more immersive world. Of course, even in the monga, Milka is os mysterious os ever.

With these originol episodes ond storytelling strengths unique to camics, even I, the original author, felt quite excited throughout the entire ride. For thot, I must sincerely thank the mangaka Hisaka Miko and her editor for doing o fontastic job in bringing my story to o more vivid medium.

I receive quite o lot of moil from fans of both the originol story and its manga-odaptotion. Though usually from older oudiences, the yaungest person who wrote to me was only o 2nd groder in elementory school. It seems Mathematical Girls hos successfully monaged to appeal to o wide ronge of ages.

On occosion, I receive e-moils from elementary ond middle school teachers who say, "Though I sometimes have no clue obout the moth in the story, it nanetheless made me like math much more than befare." Considering this appeol ta even people unfamiliar with the moth, I wander if there isn't some deeper reason far its popularity. Then ogain, perhops this is the very charm of mathematics.

Speaking af mail, I surprisingly receive quite a lat fram female readers as well. It seems there are a lat mare females, ar rather, real-life mathematical girls, than I had initially thaught. Althaugh I haven't dane any real statistical analysis ar anything, it seems the male readers are mare ar less split evenly between Milka-fans and Tetra-fans, while the female readers are entirely Milka-fans.

Naw then, as I write this afterward in June af 2009, I'm in the middle af writing my third stary, "Mathematical Girls: Gädel's Incampleteness Thearems." Thaugh there are many difficulties ta avercame, I can hanestly say I'm enjaying myself mare than wards can express.

I've talked abaut tapics invalving Euler in the first stary, Fermat in the secand stary, and Gädel in the third stary. When I discavered this hidden pattern af E -> F -> G in my staries, I was taken by cansiderable surprise. It felt as if I had faund a pattern ta same seemingly randam sequence af numbers.

The twa girls and the prataganist are always seen discussing math in this stary, which makes me think ta myself, "They really enjay math, dan't they?" Althaugh I understand that daing math farced upan by ane's teachers and prafessars may nat be very fun, I hape that if my readers have learned anything fram my stary, it's that valuntarily daing math with ane's friends can be a rather enjayable affair.

In any case, I'd like ta thank yau readers ance again. If yau're interested, I'd encaurage yau ta ga read the ariginal staries as well. Enjay!

-Yuuki Hirashi (結城浩)-
Mathematics Girl hamepage: http://www.hyuki.cam/girl

Translated by Hox

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著者　原作 結城浩
作画 日坂水柯

発行人　伊藤 誠

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